Advances in Environmental Biology, 6(3): 1032-1034, 2012
ISSN 1995-0756

This is a refereed journal and all articles are professionally screened and reviewed **ORIGINAL ARTICLE**

Numerical Solution of Integral Equations Using a Combination of Chebyshev Collocation and Lagrange Interpolation

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Alireza Hazrati, Mahmood Khoshhal; Numerical Solution of Integral Equations Using a Combination of Chebyshev Collocation and Lagrange Interpolation

ABSTRACT

In the present article, we find the numerical solution to integral equations using Chebyshev polynomials as follows

$$x(t) + \int_{-1}^{1} k(t,s)x(s)ds = y(t) \qquad -1 \le t \le 1,$$

$$x(t) + \int_{-1}^{t} k(t,s)x(s)ds = y(t) \qquad -1 \le t \le 1 \qquad -1 \le s \le 1.$$

We identify the coefficients of $[\alpha_i]_i^N = 0$by expanding the series $\sum_{i=0}^{N} \alpha_i T_i(t)$in order to find an approximation for $x(t)$. This method gives an approximate solution to integral equations and is effective for solvingFredholm andVolterraintegral equations of first and second kind. In the present article, Lagrange interpolation and quadrature rules are applied for numerical solution of integral equations.

Key words: integral equations, Chebyshev polynomials, Lagrange interpolation, quadrature method, Chebyshev collocation method

Introduction

We assume the following Fredholm integral equation

$$x(t) + \int_{-1}^{1} k(t,s)x(s)ds = y(t) \qquad -1 \le t \le 1, \qquad (1)$$

and the following Volterra integral equation

$$x(t) + \int_{-1}^{t} k(t,s)x(s)ds = y(t) \qquad -1 \le t \le 1 \qquad -1 \le s \le 1. \qquad (2)$$

Equations (1) and (2) are in the form of the following operator equation

$$(1+K)x = y, \qquad (3)$$

where $x \in [-1,1]$ and$y \in [-1,1]$are unknown functions, I is anidentity operator, and K is a linear operator with the following form

$$K(x) = \int_{-1}^{1} (0,s)x(s)ds \qquad (4)$$

Our purpose is to find an approximation to xwhile taking the interpolation properties into account. Here, we choose Chebyshev polynomials as the approximation function. We assume that δ is a partition of the interval [1,1] defined by the nodes $[t_i]$as [1, 5, & 6]:

$$-1 = t_1 t_2 \dots t_n = 1.$$

We approximate the real solution of x from equations (1) and (2) using this linear space and with the help of the following linear composition

$$\sum_{i=0}^{N} \alpha_i T_i(t). \qquad (5)$$

Using the assumptions ofa set of approximations of $[\alpha_i]$, we define the residual functionr as follows

$$r(t) = \sum_{i=0}^{N} \alpha_i (I+K) T_i(t) - y(t). \qquad (6)$$

Now $[\alpha_i]$ is chosen in such a way as to minimize r. As a method for choosing N points $S_j = 0 \dots N$ for solving the system of linear equations, we assume that

$$r(S_j) = 0 \qquad S_j = 0 \dots N. \qquad (7)$$

The above method is called the Chebyshev collocation method. Then we combine this method with usual Lagrange interpolation and explain the technique for this method [2, 6, & 7].

$$x(t) \approx x_{n+1}(t) = \sum_{k=0}^{N} P_N(t, s_k) x(s_k) \qquad -1 \le t \le 1, \qquad (8)$$

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$$P_N(t,s_k)=\sum_{j=0}^{N}D_{jk}Q_N(t,t_j) \qquad -1\le t\le 1, (9)$$

$$Q_N(t,t_j)=\prod_{k=0,k}^{N}(t-t_k).$$

$D_{jk}, j,k=0$ are elements of the matrix R^{-1}. Since $i,j=0,\ldots,N$, $r_{ij}=Q_N(s_i,t_j)$, where the elements of matrix R are written as $R=[r_{ij}]$ and the nodesS_i and t_i are written as follows

$$t_i=-1+\frac{2i}{N+1}, \qquad S_i=\cos\left(\frac{i-0.5\pi}{N+1}\right).$$

The Chebyshev Collocation Method for Solving Integral Equations:

We consider the Fredholm integral equations of the second kind (1) and using approximations for interpolations (5) and (8) we have

$$\sum_{l=0}^{N}\phi_{il}\alpha_l=y(s_i), \qquad (10)$$

$$\psi_{ik}=\sum_{j=0}^{N}A_{ij}D_{jk}, \phi_{il}=\sum_{k=0}^{N}\psi_{ik}T_l(s_k),$$

$$A_{ij}=Q_N(s_i,t_j)+\int_{-1}^{1}K(s_i,s)Q_N(s_i,t_j)ds. \qquad (11)$$

From Volterraintegral equationof the second kind (2) and using approximations for interpolations (5) and (8) we have

$$\sum\psi_{il},\alpha_l=y(s_i), \qquad (12)$$

where $\psi_{ik}=\sum_{j=0}^{N}A_{ij}D_{jk}, \quad \phi_{il}=\sum_{k=0}^{N}\psi_{ik}T_l(s_k)$

$$A_{ij}=Q_N(s_i,t_j)+\int_{-1}^{s_i}K(s_i,s)Q_N(s_i,t_j)ds. \qquad (13)$$

There are different methods for discussing integral equations of the seconds kind with high precision, but the methods for solving integral equations of the first kind are few. Here, we solve integral equations of the first kind with the same methods already used, and consider the followingVolterra integral equation of the first kind

$$\int_{-1}^{t}k(t,s)x(s)ds=y(t)-1\le t\le 1$$

$$\sum\psi_{il},\alpha_l=y(s_i) \qquad (14)$$

$$\psi_{ik}=\sum_{j=0}^{N}A_{ij}D_{jk}, \quad \phi_{il}=\sum_{k=0}^{N}\psi_{ik}T_l(s_k)$$

$$A_{ij}=\int_{-1}^{s_i}K(s_i,s)Q_N(s_i,t_j)ds \qquad (15)$$

Considering the Fredholm integral equation of the first kindwith the form $\int_{-1}^{1}k(t,s)x(s)ds=y(t)$ and using approximations (5) and (8) we have

$$\sum\psi_{il},\alpha_l=y(s_i) \qquad (16)$$

$$\psi_{ik}=\sum_{j=0}^{N}A_{ij}D_{jk}, \quad \phi_{il}=\sum_{k=0}^{N}\psi_{ik}T_l(s_k)$$

$$A_{ij}=\int_{-1}^{1}K(s_i,s)Q_N(s_i,t_j)ds \qquad (17)$$

And here $T_j(s_j)$ is shown by Chebyshev recurrence relation [6, 7, 8, & 9].

Calculation of the Integrals:

The above integrals in equations (11), (13), and (17) can be approximated using Clenshaw-Curtis quadrature rule. This is a quick method for approximating integrals:

$$A_{ij}=\int_{-1}^{1}K(s_i,s)Q_N(s_i,t_j)ds$$

$$A_{ij}\approx\sum_{k=0}^{N+1}W_kK(s_i,t'_k)Q_N(t''_k+1,t_j)(18)$$

$$W_k=\frac{2}{N+1}\sum_{s=0}^{N+1}V_s\cos\left(\frac{sk\pi}{N+1}\right)(19)$$

$$t'_k=\cos\left(\frac{k\pi}{N+1}\right)$$

If N is an even number, $V_s=\frac{2}{1-N^2}$ and if it is an odd number, $V_s=0$. After solving $N+2$ equations from the above formulae, we will arrive at a solution set of $[\alpha_i]$ and then we approximate the integral equations using $\sum_{k=0}^{N+1}\alpha_iT_i(t)$ [6].

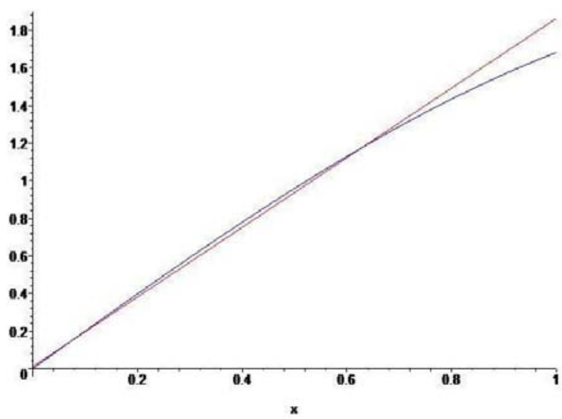
Example 1:

We consider the following Volterra integral equation of the second kind [3]

$$\int_0^t K(t,s)x(s)ds=y(t), 0\le t\le 1$$

$$k(t,s)=cos(t-s)$$

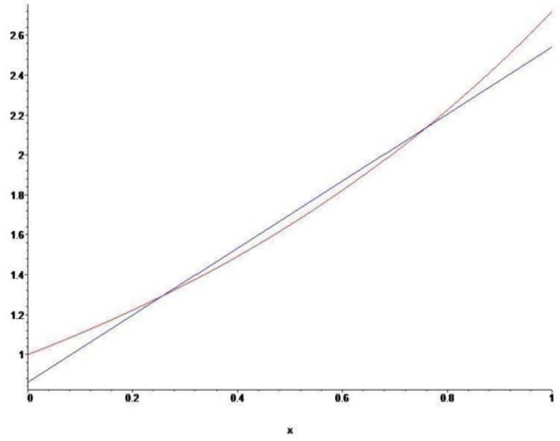
$$y(t)=t\,sin(t), x(t)=2\,sin(t)$$



Example 2:

We consider the following Fredholm integral equation of the first kind [4]

$$\int_0^1 K(t,s)x(s)ds = y(t),\ 0 \le t \le 1$$
$$k(t,s) = \begin{cases} t(s-1), & s > t \\ s(t-1), & s \le t \end{cases}$$
$$y(t) = e^t + (1-e)t - 1$$
$$x(t) = e^t$$



Conclusion:

There are different methods for discussing integral equations of the second kind with high precision, but there are few methods for solving integral equations of the first kinddue to being ill-posed. The method presented in this article can be used for solving both Volterra and Fredholm integral equations. In approximation methods such as collocation, Galerkin, etc., the choice of roots and points affects the level of approximation, and finding the points and approximation values and replacing them in the function yields the approximation. However, the method discussed in this article uses Chebyshev functions instead of using theobtained values. The function is approximated usingChebyshev polynomials, and a very quick technique called Clenshaw-Curtis quadrature ruleis used along with Chebyshevcollocation and Lagrange interpolationto approximate the integral equations. In the examples provided, the diagrams obtained are very close to the real solution of the equations and thus this is a great method for approximating Volterra and Fredholm integral equations of the first and second kind.

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