

Correction on Chang *et al.* equation factors of suspended load estimation (Case Study: Central Alborz Rivers of Iran)**¹A. Salajegheh, ²Y. Kazemi, ³N. Rostami, ⁴M.M. Heidari**¹ *Associated Professor, Natural Resources Faculty, University of Tehran, Iran*² *M.Sc. of Watershed Management, University of Tehran, Iran*³ *Ph.D. candidate of Watershed Management sciences and engineering, University of Tehran, Iran*⁴ *Ph.D. student of Irrigation and Reclamation engineering Department, University of Tehran, Iran*A. Salajegheh, Y. Kazemi, N. Rostami, M.M. Heidari; Correction on Chang *et al.* equation factors of suspended load estimation (Case Study: Central Alborz Rivers of Iran)**ABSTRACT**

The subject of sediment transport has been noticed by engineers for years. Several methods had been applied for sediment issues whose results are various empirical equation offered by experts. Every of these equations are gained based on climate or laboratory conditions in different places in the world and they have different factors for determining transporting sediment amount, therefore, in most of these equations some graph and equations are offered by researchers for simplifying various factors. Evaluation of I_1 and I_2 factors in Chang *et al.* (1967) equation in different references showed some errors on it. After scrutiny of different references and use of FORTRAN program, integrals and graphs of I_1 and I_2 factors of this equation are corrected.

Key words: Chang *et al.* (1967) equation, central Alborz Rivers, empirical equations, Sediment transport, Iran.**Introduction**

Yang [5]: Rivers always have erosion and sediment transport, thus in river hydraulic and its morphology, survey of sediment transport capacity of stream and sediment transport mechanism are of high importance. Sediment of rivers transported in two general forms: suspended load and bed load and in most of natural rivers, sediment mostly transported as suspended load.

Shafai Bajestan and Ostad Asgari [3]: because in most of the rivers there is not any sedimentography and hydrometry station for sediment evaluation and also because of difficulty in sediment measurement, and need to experts and costly professional equipments, researchers offered various equation for simpler and better measurement of sediment in rivers and channels, in the recent years.

Some of these methods evaluate just suspended load, some do only bed load and some others evaluate collection of them, called bed materials. The majority of these methods are formed based on laboratory works and always their application is in question.

Shafai Bajestan [2]: there are different equation for suspended load determination such as Lane and Kalinske, Einstein, Brooks, Chang, Simons and Richardson, Bagnold, Toffaleti, Samaga and so on.

Any of these equations have various factors for determining suspended sediment amount and in most of the cases researchers offer graphs and integrals for simplifying these factors among which are Einstein, Toffaleti and Chang *et al.*

The goal of this study is checking the authenticity of I_1 and I_2 factors in Chang *et al.* equation and also offering the best integral and graph for I_1 and I_2 factors on this equation.

Materials and methods

In this study for correction of Chang *et al.* factors, data of Central Alborz Rivers are used.

Chang, Simons and Richardson (1965):

By applying the velocity distribution $\left(\frac{du}{d\xi} = U_* / K \xi \sqrt{1 - \xi} \right)$ over the depth, Chang *et al.* obtained the following expression for the mass transfer coefficient:

$$\varepsilon_s = \beta K D \xi U_* (1 - \xi)^{1/2} \quad \text{Eq.1}$$

Where, $\xi = y/D$ and $(U_* = (gds)^{1/2})$. Substituting Eq.1 into Eq.2

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$$C = C_a \exp\left(-\omega \int_a^y \frac{dy}{\varepsilon_s}\right) \tag{Eq.2}$$

Where, C_a and C =concentration by weight of sediment in (a) and (y) distances above bed respectively and it obtained from Rouse equation.

$$\frac{C}{C_a} = A_1 \left[\frac{\xi^{1/2}}{1 - (1 - \xi_a)^{1/2}} \right]^{Z_2} \tag{Eq.3}$$

In this relation, $A_1 = \left(\frac{1 - \sqrt{1 - \xi_a}}{\sqrt{\xi_a}} \right)^{Z_2}$
 $Z_2 = 2W/(\beta U_* K)$ and $\xi_a = a/d$.

Then the expression for the suspended load becomes:

$$q_{sw} = \int_a^d C \omega dy = d C_a \left(U I_1 - \frac{2U_*}{K} I_2 \right) \tag{Eq.4}$$

Where, I_1 and I_2 are integrals that can be evaluated using figs.1 and 2. The transport q_{sw} is measured in weight per volume of water-sediment mixture.

If q_{sw} is expressed in weight per second per unit channel width and C_a is concentration by weight, then:

$$q_{sw} = \gamma D C_a \left(V I_1 - \frac{2U_*}{k} I_2 \right) \tag{Eq.5}$$

Examining Chang *et al.* equation:

For estimation of suspended load in a river by Chang *et al.* equation, we need to extract I_1 and I_2 factors by use of relative graph or integral and substituting factors in equation.

Kazemi [1]: when data are low, determining the numeral amount of I_1 and I_2 factors by corresponding Graphs is simply possible, but in this study which is, applied for estimation of suspended load by Chang method in rivers of Central Alborz of Iran, for estimation of suspended load in Jajrod river, the stream simulation was done in 17 cross section of this river and 30-year annual average discharge was used. Totally about 1020 data for selection of I_1 and I_2 was obtained. Also in Taleghan river simulation was done in 25 cross section and 30 years annual average discharge was used. Totally about 1500 data for selection of I_1 and I_2 was obtained. Finding these data by graphs need a lot of time and thereupon amount of error will increase.

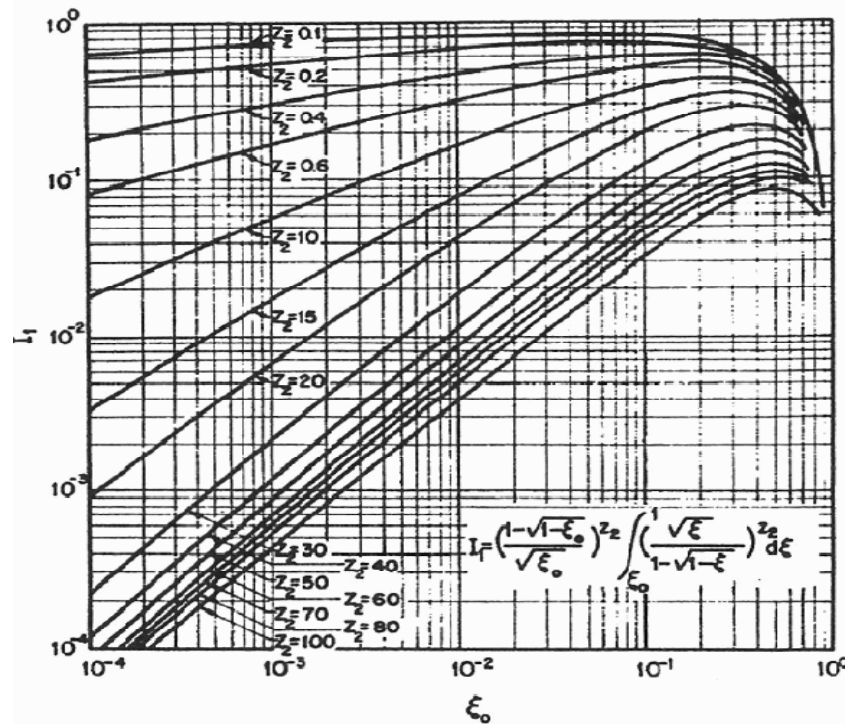


Fig. 1: The function I_1 in terms of ξa for various values of the exponent Z_2

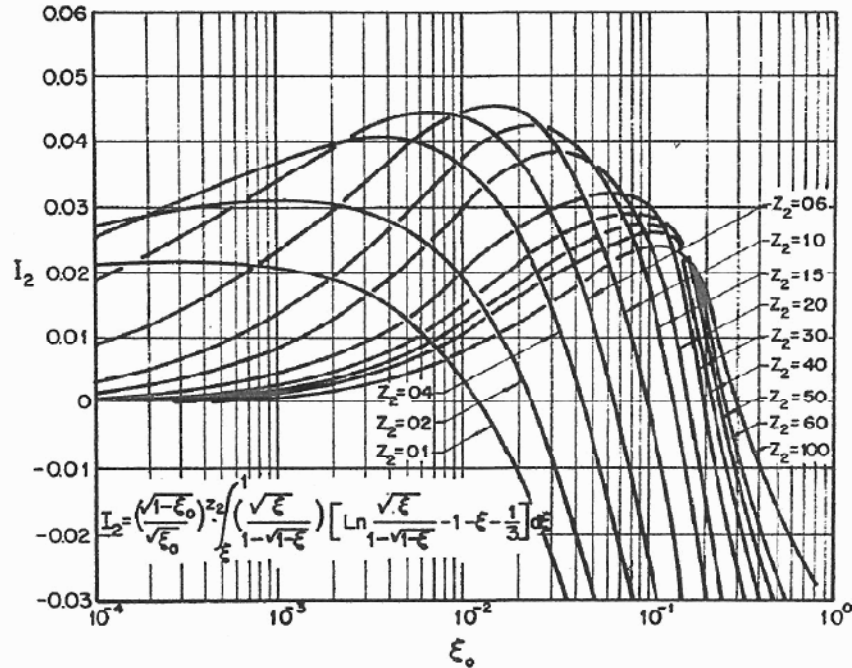


Fig. 2: The function I_2 in terms of ξa for various values of the exponent Z_2

Hence, for solving this problem it was suggested that numeral solution of integrals of these factors be used instead of graphs. Therefore, a program was written by FORTRAN language and use of Yang reference to obtain numeral amount of integrals by entering required parameters of I_1 and I_2 factors. After writing FORTRAN program, for assurance about the program accuracy, some data were entered in the program and at the same time the answer of data was determined by use of graphs. Comparison between output of FORTRAN program and graphs showed that there is an incompatibility between them. After assurance about authenticity of written program, error Possibility in offered graphs and integrals in different references were examined. For this reason, first Yang reference and then older references of this equation (Simons and Senturk reference) were examined.

Results:

Survey of two mentioned references about Chang *et al.* equation showed that there are some errors in graphs and integrals of this equation.

Examining graphs and integrals of I_1 factor in Simons and Senturk reference:

Examining I_1 factor in Simons and Senturk reference showed that integral of this factor (Eq.6) is correspondent with output of Fortran program but checking of graphs of this factor (fig.3) showed that Z_2 value in this graph that started from 0.1 is correct

until 0.6 but from 0.6 to end, Z_2 value has error and every of this value is presented 10 times its actual value.

Examining I_1 factor in Yang reference showed that the integral of this factor has some errors (Eq.7) but the presented graph in this reference is correct (fig.4).

I_1 factor in Simons and Senturk reference:

$$I_1 = \left[\frac{1 - (1 - \xi/a)^{1/2}}{\xi/a} \right]^{Z_2} \int_{\xi/a}^1 \left[\frac{\xi^{1/2}}{1 - (1 - \xi/a)^{1/2}} \right]^{Z_2} d\xi \tag{Eq.6}$$

I_1 factor in Yang reference:

$$I_1 = \left[\frac{1 - (1 - \xi/a)^{1/2}}{\xi/a^{1/2}} \right]^{Z_2} \int_{\xi/a}^1 \left[\frac{\xi^{1/2}}{1 - (1 - \xi)^{1/2}} \right]^{Z_2} d\xi \tag{Eq.7}$$

Examining graphs and integrals of I_2 factor in Simons and Senturk reference:

For checking I_2 factor a comparison was also done between mentioned references and written program by FORTRAN. Scrutiny showed that the considered Integrals in both of references have some error (eq.8 and 9), also checking graphs showed that graph of Simons and Senturk reference (fig.5) has error and graph of Yang reference (fig.6) is correct.

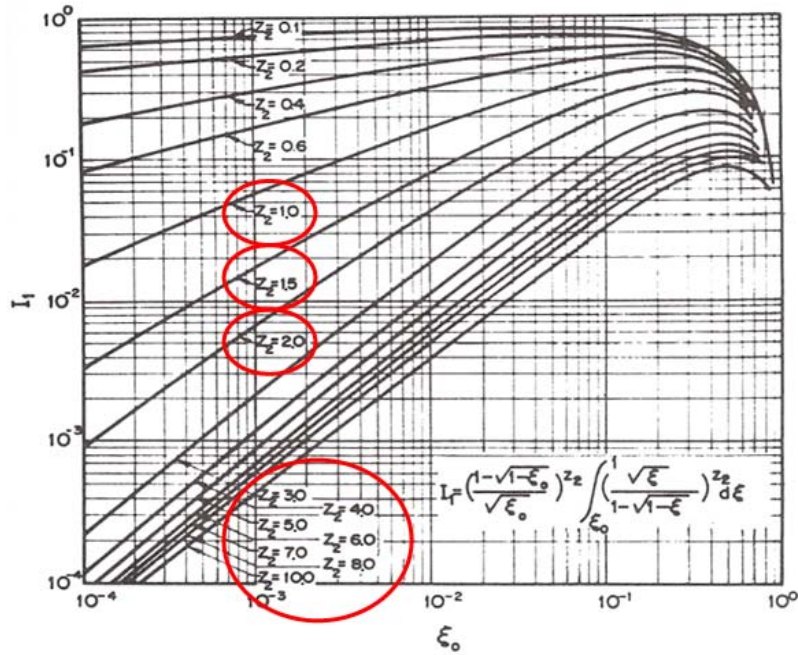


Fig. 3: I₁ factor in Simons and Senturk reference.

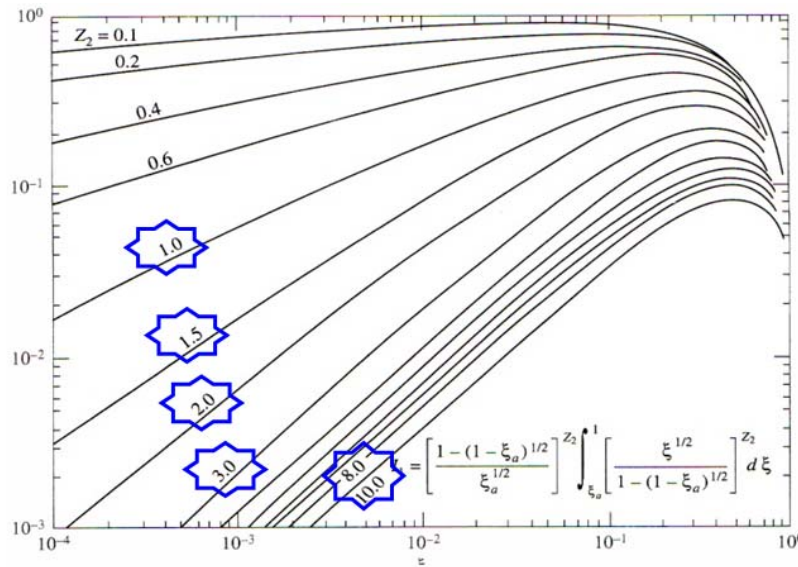


Fig. 4: I₁ factor in Yang reference.

I₂ integral in Simons and Senturk reference

$$I_2 = \left(\frac{\sqrt{1-\xi_0}}{\sqrt{\xi_0}} \right)^{Z_2} \int_{\xi_0}^1 \left(\frac{\sqrt{\xi}}{1-\sqrt{1-\xi}} \right)^{Z_2} \ln \left(\frac{\sqrt{\xi}}{1-\sqrt{1-\xi}} - 1 - \xi - \frac{1}{3} \right) d\xi$$

Eq.8

I₂ integral in Yang reference

$$I_2 = \left[\frac{1-(1-\xi_a)^{1/2}}{\xi_a^{1/2}} \right]^{Z_2} \int_{\xi_a}^1 \left(\frac{\xi}{1-\xi} \right)^{Z_2} \left[\ln \frac{\xi^{1/2}}{1-(1-\xi)^{1/2}} - (1-\xi)^{1/2} - \frac{1}{3} \right] d\xi$$

Eq.9

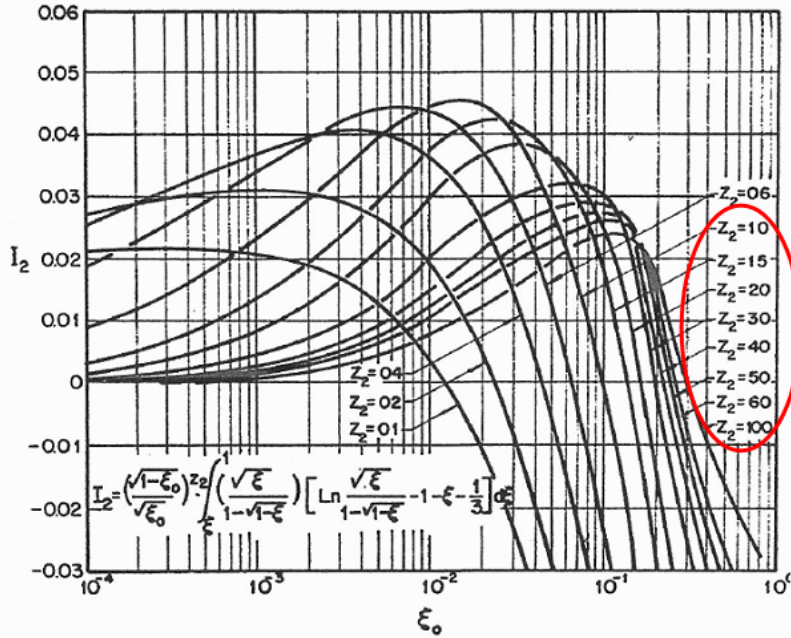


Fig. 5: I₂ factor Simons and Senturk reference.

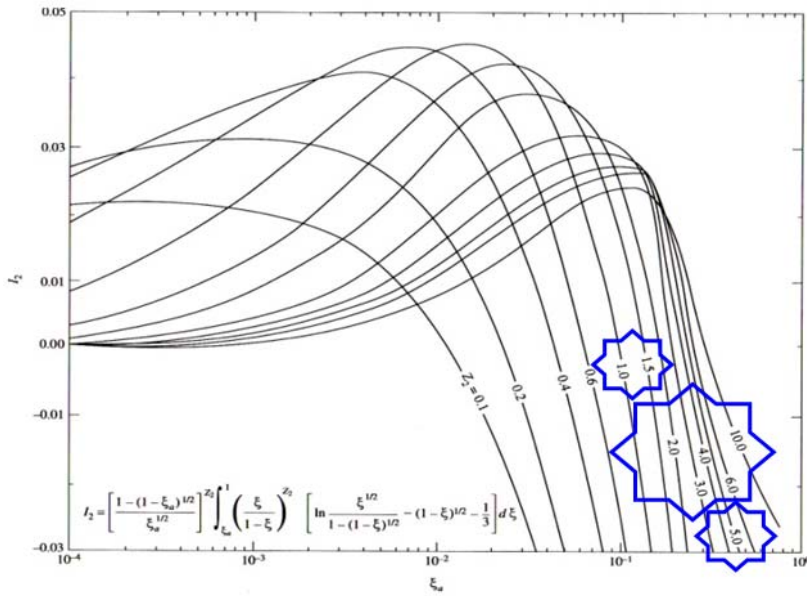


Fig. 6: I₂ factor in Yang reference.

Selection of best graph and integral for I₁ and I₂ factors:

After checking mentioned references and comparing them with written program by FORTRAN it was clear that integral of Simons and Senturk reference (Eq.6) and graph of Yang reference (Eq.4) offer best answer for I₁ factor.

For I₂ factor examines showed that graph of Yang book (fig.6) offer the best answer but none of

relative integrals have accordance with FORTRAN program.

Therefore, all of status were first written for integral of I₂, and then by FORTRAN program different status of integral of I₂ factor were obtained. Comparing the results of FORTRAN program with graphs data showed that the following equation (Eq.10) has the best answer for I₂ factor.

$$I_2 = \left(\frac{1 - \sqrt{1 - \xi_0}}{\sqrt{\xi_0}} \right)^2 \int_{\xi_0}^1 \left(\frac{\sqrt{\xi}}{1 - \sqrt{1 - \xi}} \right)^2 \ln \left(\frac{\sqrt{\xi}}{1 - \sqrt{1 - \xi}} - (1 - \xi)^{1/2} - 1/3 \right) d\xi$$

Eq.10

Conclusion:

Solving some of the sediment transport equations like those of Einstein, Chang, Simons – Richardson and Toffaleti for estimation of sediment transport amount, needs extracting many of relative factors on these equations by use of graphs and diagrams of these equations or solving difficult integrals of these equations.

Obtaining different factors by use of graph and diagram just is possible when data are a few is possible and when data are a lot, integrals of these graphs must be solved. Studying different references shows error in graphs and integrals of sediment transport equations and in many cases a sediment transport equation is offered in different forms in various references and it causes error in sediment estimation by use of these equations.

Studying Chang *et al.* equation in different references showed that integral and graph of I_1 and I_2 factors of this equation in various references have some errors. These errors are corrected by use of a program written in FORTRAN language and the best graph and integral for I_1 and I_2 factors of Chang *et al.* equation were obtained.

Results of this study clearly show the possibility of error in different equation for sediment transport estimation. The result of error on these equations leads to error on actual sediment amount.

Based on result of this study, we suggest being careful in using the sediment transport estimation and it will be better to use the main reference of these equations. We also suggest using different computer programs which are capable of solving difficult equations, simulation of diagrams and graphs of sediment transport equations to be sure about the authenticity of various equations.

Notation:

I_1 and I_2 : parameters in Chang's transport equation

U_* : shear velocity

K: constant

ξ : Relative depth = $\frac{y}{D}$

ε_s : momentum diffusion coefficient for sediment

β : Constant

D : average flow depth

g : gravitational

d : sediment particle diameter

s : water surface or energy slope; or slope

C : sediment concentration

C_a : sediment concentration

ω : fall velocity of sediment particle

a : thicknesses of bed layers

u : local velocities

γ : Specific weight of water

k : Von Karman –Prandtl universal constant (= 0.4)

V : average flow velocity

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