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Frequency Optimization of Conical Shells under Mass Equality Constraint

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Abstract: As a major structural dynamic criterion in designing thin shells, the fundamental frequency is maximized for a specified total structural mass. The general optimization problem of metallic conical shell structures with clamped / free ends has been treated formally among all other types of boundary conditions. Both stiffened and unstiffened constructions are examined with the effective design variables selected to be the shell thickness distribution and the number, locations and size of the attached ring stiffeners. Structural analysis is based upon Donnell-Mushtari shell theory and an analytical approach based on Rayleigh-Ritz method has been implemented for calculating the natural vibration characteristics of the different shell types. The final optimization problem is formulated as a nonlinear mathematical programming problem solved by invoking the Matlab optimization toolbox routines, which implement the interior penalty function technique and interact with eigenvalue calculation routines. The proposed mathematical model has succeeded in maximizing the fundamental frequency without the penalty of increasing the total structural mass. Results show that the approach used in this study is efficient and produces designs having improved dynamic performance as compared with a known baseline design.

Key words: Thin shells, structural dynamics, design optimization.

INTRODUCTION

Shell structures have been widely used in many applications of civil, aerospace and marine engineering. Their design involves many advantages, such as light structural mass, high stiffness level, long fatigue life and low vibration levels. Minimal mass is of paramount importance for successful design, as it improves the overall performance characteristics and reduces the cost of production. Adequate stiffness is essential to enhance the stability in the face of the violent interaction between aerodynamic, inertia and elastic forces. Moreover, high stiffness levels decrease the chance of fatigue failure under cyclic loading. Another cost-effective solution for a successful shell design is the minimization of the overall vibration level, which fosters other important design goals, such as long fatigue life and low noise levels. Maximization of the natural frequencies is a good measure of vibration reduction, which is favorable for decreasing both of the steady state and transient response of the structure being excited. Natural frequencies are the actual measure of the stiffness-to-mass ratio of a structural member. Their maximization is a more straightforward design criterion than maximization of the stiffness alone or minimization of the structural mass alone.

The available literature dealing with vibration of shells shows that great efforts have been devoted to this area although; much work is still needed to cover some other aspects. Leissa[1] provided an exhaustive and thorough review of vibration analysis of shells that includes a survey of numerical solutions and comparisons of various shell theories. The equations of motion for circular cylindrical and conical shells were studied by Soedel[2], where approximate techniques based on Rayleigh-Ritz principle were used for the solution of the associated eigenvalue problems with different boundary conditions. Considering stiffened shells, Ref.[3] studied vibration characteristics of conical shells numerically within the context of Donell-Mushtari theory. The natural frequencies of clamped-free shells were also calculated by Rayleigh Ritz method. The study showed that the rings rather than the stringers raise the fundamental frequency considerably because the circumferential stiffness of the shell affects its dynamic behavior more than the meridional stiffness. Wang *et al*.[4] have also used the Ritz polynomial functions for the axial modal dependence to analyze ring-stiffened cylindrical shells. These appeared to provide a good convergence to the Rayleigh-Ritz method. Irie *et al*.[5] studied the free vibration of truncated conical shells with variable thickness by the transfer matrix approach for a given set of boundary conditions. Mecitoglu[6] was also concerned with the free vibrations of stiffened conical shells within the context of Donnell-Mushtari theory. The equations of motion of a truncated cone reinforced by relatively closely spaced elastic stringers with simply supported ends were derived by the use Hamilton's principle. The collocation method was implemented to solve the associated eigenvalue problem.

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A great deal of research on structural dynamic optimization of beams[7], plates and shells[8-11], has been conducted during the past three decades. Most of the work treated the dynamic frequency as a design constraint. Ref.[7] used standard non-gradient methods to avoid the singularities in calculating the eigenvalue derivatives with respect to the design variables due to the multiplicity of the objective function, which was measured by maximization of a weighted-sum of the system natural frequencies. Optimization of axisymmetric thin-shallow shells for maximum fundamental frequency was considered by Bratus[8]. The boundary conditions, material, surface area and uniform thickness of the shell were specified as constraints. Bimodal formulation was used and an iterative procedure based on the optimality condition was implemented. Akl *et al.*[9] presented a process for the design of stiffened shell structures subjected to hydrostatic pressure. Design variables included the number of stiffeners and their cross sectional dimensions. A multi-criteria optimization approach was utilized to select the optimal values of the design variables that minimize the maximum amplitude of vibration, sound intensity level and weight of rings, as well as the total cost of the structure. Another work[10] dealt with minimal weight design of stiffened shell structures, where an evolution strategies method was implemented for the solution of the optimization problem coupled with finite element structural analysis.

It is the major aim of the present study to investigate the natural frequencies of a metallic shell structure having general conical shape and formulate an appropriate design optimization model aiming at the reduction of the overall vibration level without the penalty of increasing structural mass. The paper contains seven sections. Section two introduces a brief definition of conical shell structure and the mathematical model description. Structural analysis and the associated eigenvalue problem of isotropic, stiffened conical shells are covered in sections three and four. The optimization model formulation is given in section five, showing the selected design variables, constraints and preassigned parameters. Section six presents the attained optimal solutions for the different shell configurations. Finally, the future aspects and conclusions are drawn in section seven.

2. Model Description: The general case of a stiffened metallic conical shell is considered to define the different geometrical relationships. Fig. 1 shows truncated, homogeneous, isotropic, shell models having either divergent or convergent configuration. The coordinate system is defined as follows: s is measured along the cone generator starting at cone vertex, θ is the circumferential coordinate and z is the radial coordinate. The semi- vertex angle of the cone α and the distance s are defined by the relations:

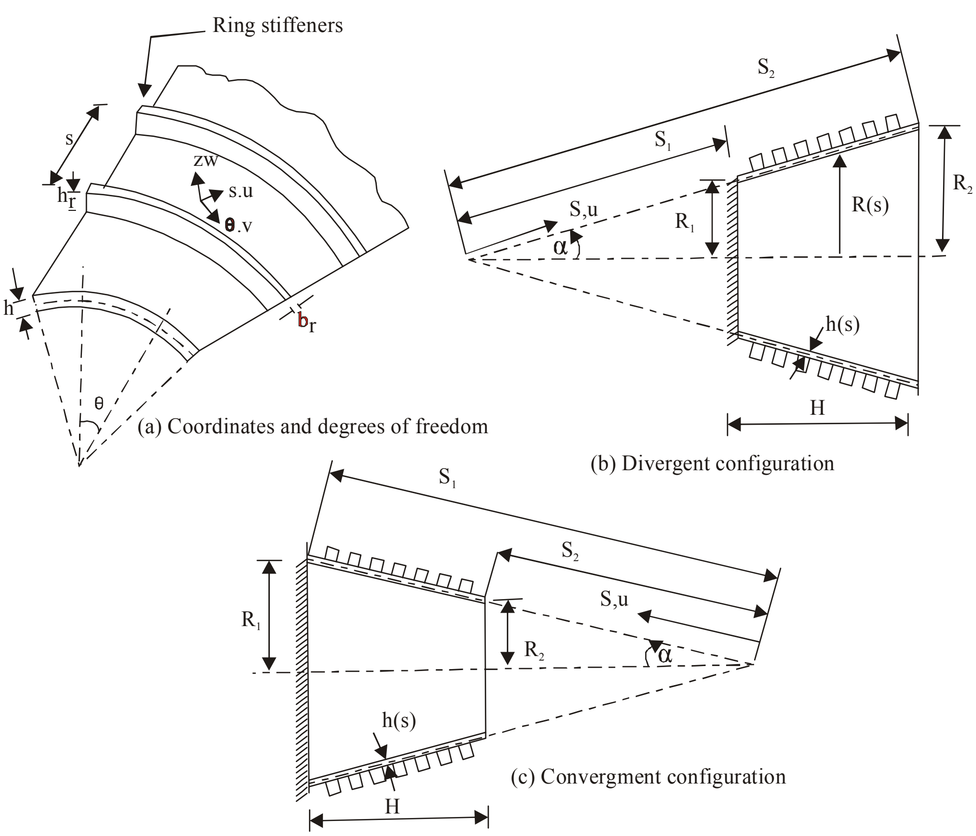


Fig. 1: General configuration of clamped/free stiffened conical shells.

$$\tan\alpha = (R_2-R_1)/H \text{ and } s = R(s)/\sin\alpha \tag{1}$$

where H is the height of the cone, R_1 and R_2 are the radii at the clamped and free ends, respectively. The variation of the shell thickness, *h(s)*, is considered as a power function expressed by the relation:

$$h(s) = h_1 + (h_2 - h_1)\left(\frac{s-s_1}{L}\right)^m \tag{2a}$$

where h_1 and h_2 are the shell thicknesses at the clamped and free edges, respectively. Shells with uniform, linear or parabolic thickness distribution shall have values of the integer exponent m equal to 0, 1 or 2, respectively. A more general case with quadratic thickness distribution is:

$$h(s) = h_1 - (3h_1 - 4h_3 + h_2)\left(\frac{S-S_1}{L}\right) + (2h_1 - 4h_3 + 2h_2)\left(\frac{S-S_1}{L}\right)^2 \tag{2b}$$

where h_3 is the thickness at the middle section.

3. Energy Formulation: Structural analysis in the present study is based upon Donnell-Mushtari shell theory[3,6]. The equations for the strain and kinetic energies for the general case of stiffened shell are integrated symbolically by using mathematica program.

3.1 Strain Energy: The total strain Energy of a ring stiffened conical shell may be written as follows:

$$U = U_c + U_r \tag{3}$$

where U_c is the stored strain energy in the conical shell alone and U_r is that in the stiffeners alone. The former can be obtained from the integral expression:

$$U_c = \tfrac{1}{2}\int_{s_1}^{s_2}\int_{0}^{2\pi}\{C[\varepsilon_s^2 + \varepsilon_\theta^2 + 2\nu_c\varepsilon_s\varepsilon_\theta + \frac{1}{2}(1-\nu_c)\ \varepsilon_{s\theta}^2] + D[\kappa_s^2 + \kappa_\theta^2 + 2\nu_c\kappa_s\kappa_\theta + 2(1-\nu_c)\kappa_{s\theta}^2]\}\, R(s)\, d\theta\, ds \tag{4}$$

The parameters C and D denote the stretching and bending rigidities of the shell;

$$C = \frac{E_c h}{1-\nu_c^2} \text{ and } D = \frac{E_c h^3}{12(1-\nu_c^2)} \tag{5}$$

E_c and ν_c are the Young's modulus and Poisson's ratio of the shell material, respectively. In accordance with Donnell–Mushtari conical shell theory[6], the strain-displacement and curvature-displacement relations are:

$$\varepsilon_s = \frac{\partial u}{\partial s},\quad \varepsilon_\theta = \frac{u}{s} + \frac{1}{R}\frac{\partial v}{\partial\theta} + \frac{w}{R}\cos\alpha,\quad \varepsilon_{s\theta} = \frac{1}{R}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial s} - \frac{v}{s} \tag{6}$$

$$\kappa_s = -\frac{\partial^2 w}{\partial s^2},\quad \kappa_\theta = -\frac{1}{R^2}\frac{\partial^2 w}{\partial\theta^2} - \frac{1}{s}\frac{\partial w}{\partial s},\quad \kappa_{s\theta} = -\frac{1}{R}\frac{\partial^2 w}{\partial s\,\partial\theta} + \frac{1}{R^2}\frac{\partial w}{\partial\theta} \tag{7}$$

where u, v and w are the components of displacement at the middle surface in the s, θ and z directions, respectively. It is to be noticed that, according to Love's hypotheses for linear elastic shells[1-3], a linearly distributed tangential displacement and a constant normal displacement through the thickness of the shell were postulated, which nullifies the shear strain components ε_{SZ} and $\varepsilon_{\theta Z}$. On the other hand, the contribution from the stiffeners, U_r is given by:

$$U_r = \tfrac{1}{2}\sum_{r=1}^{N_r}\left(\int_0^{2\pi}\{E_r A_r[\varepsilon_\theta^2 - 2c_r\varepsilon_\theta\kappa_\theta + (i_r^2 + c_r^2)\,\kappa_\theta^2] + G_r j_r \kappa_{s\theta}^2\} R_r\, d\theta\right)_r \tag{8}$$

where N_r is the total number of ring stiffeners, E_r the Young's modules and G_r the modules of rigidity of the stiffeners material. A_r, J_r, c_r, i_r and R_r are the cross sectional area, torisonal constant, distance to the centroid of the cross section from the shell middle surface, radius of gyration and the radius of the rth stiffener, respectively.

3.2 Kinetic Energy: Neglecting the effect of rotary inertia, the contribution of the conical shell alone to the total Kinetic energy is:

$$T_c = \frac{1}{2}\int_{s_1}^{s_2}\int_0^{2\pi}\rho_c h(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)R(s)\,d\theta\, ds \tag{9}$$

where ρ_c is the mass density of the shell material and the dot denotes time derivative. The effect of the rotary inertia of the stiffeners may also be neglected for lower modes of vibration. Thus, the kinetic energy of the ring stiffeners is:

$$T_r = \frac{1}{2}\sum_{r=1}^{N_r} \left(\int_0^{2\pi} \{\rho_r A_r (\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\} R_r \, d\theta . ds \right)_r \quad (10)$$

The total kinetic energy of the whole stiffened conical shell is then given by adding both contributions, i.e.

$$T = T_c + T_r \quad (11)$$

4. The Eigenvalue Problem: The natural frequencies have been obtained by Ralyeigh-Ritz method[3,4], where a sequence of approximating solutions is constructed from the space of either admissible functions or comparison functions. The admissible functions need satisfy only the geometric boundary conditions, whereas the comparison functions must satisfy all the boundary conditions.

4.1 Boundary Conditions: For clamped–free conical shell, the natural boundary conditions are:

$$u = v = w = \frac{\partial w}{\partial s} = 0 \text{ at } s = s_1 \text{ (clamped end)} \quad (12)$$

and the static boundary conditions are:

$$N_s = M_s = S_{s\theta} = V_s = 0, \quad \text{at } s = s_2 \text{ (free end)} \quad (13a)$$

where

$$S_{s\theta} = N_{s\theta} + \frac{M_{s\theta}}{R}\cos\alpha \quad (13b)$$

$$V_s = \frac{1}{R}\frac{\partial M_{s\theta}}{\partial \theta} \quad (13c)$$

The forces and moments related to the membrane and bending strains are obtained by the following relations:

$$N_s = C(\varepsilon_s + \nu\varepsilon_\theta),$$
$$N_\theta = C(\varepsilon_\theta + \nu\varepsilon_s),$$
$$N_{s\theta} = N_{\theta s} = 4Gh\,\varepsilon_{s\theta}$$

$$M_s = D(\kappa_s + \nu\kappa_\theta),$$
$$M_\theta = D(\kappa_\theta + \nu\kappa_s),$$
$$M_{s\theta} = M_{\theta s} = \frac{1}{6}(G_c h^3)\ \kappa_{s\theta} \quad (14)$$

Using Rayleigh–Ritz method, it is sufficient that the natural boundary conditions are fulfilled and the arbitrary constants are then used to fulfill the static boundary conditions as far as possible[3].

4.2 Mode Shape Assumption: Since the modeshapes are orthogonal, the displacements u, v and w may be taken as polynomials satisfying the natural boundary conditions. For the case of divergent conical shells, they were chosen to have the mathematical form:

$$u = (s - s_1)\ (C_1 s^{-2} + C_2 s^{-1} + C_3)\ \sin(n\theta)\cos(\omega t)$$

$$v = (s - s_1)\ (C_4 s^{-2} + C_5 s^{-1} + C_6)\ \cos(n\theta)\cos(\omega t)$$

$$W = (s - s_1)^2\ (C_7 s^{-2} + C_8 s^{-1} + C_9)\ \sin(n\theta)\cos(\omega t) \quad (15)$$

where C_i, i=1,2,….9 are arbitrary constants. The same equations are applied to the case of convergent shell by replacing $(s - s_1)$ by $(s_1 - s)$.

4.3 Frequency Equation: The assumed displacements are substituted into the expressions of the strain and kinetic energies. This leads to the following compact expression:

$$U = U_{max}.\cos^2(\omega t)$$
$$T = T_{max}.\sin^2(\omega t) \quad (16)$$

For a conservative system, the maximum kinetic energy, T_{max}, must be the same as the maximum strain energy, U_{max}. The arbitrary set of independent constants C_i will be chosen such that the difference between kinetic and strain energies is as small as possible, i.e.

$$\frac{\partial (T_{max} - U_{max})}{\partial C_i} = 0, \quad i = 1, 2, ...9 \quad (17)$$

This leads to the following set of linear equations:

$$\{[p] + \omega^2 [Q]\}[C] = [0] \quad (18)$$

where [p] and [Q] are 9x9 square matrices and [C] is the vector of the arbitrary constants C_i. The problem of finding the natural frequencies is now reduced to an eignvalue problem. Implementation to Mathematica[12] is rather straightforward. It is one of the most powerful software machine programs that can carry out the most difficult symbolic integrations. The energy integrals are first symbolically integrated in the circumferential direction and then the obtained results are substituted into the derived expressions. The remaining meridional integration is done numerically for every value of the circumferential wave number n.

5. Optimizaion Problem Formulation: The present formulation seeks maximization of the fundamental (lowest) frequency of the shell with the total structural mass kept at a fixed value equal to that of a known baseline design. The total structural mass, M, is given by

$$M = M_c + M_r \tag{19}$$

where M_c is the mass of the conical shell given by

$$M_c = \int_{s_1}^{s_2} \rho_c \; 2\pi R \; h \; ds \tag{20}$$

Considering ring stiffeners with solid square cross section, the total mass of the stiffeners M_r is:

$$M_r \cong \sum_{k=1}^{Nr} \rho_r 2\pi R_{r,k} b_{r,k}^2 , \; R_{r,k} = S_{r,k} \sin\alpha \tag{21}$$

where $S_{r,k}$ and $b_{r,k}$ are the location and width of the kth ring stiffener, respectively. In the above equation, it is assumed that $\frac{(h + b_{r,k})}{2R_{r,k}} \langle\langle \; 1.0$

5.1 Design Variables: The design variables which are subject to change in the optimization process are chosen to be the wall thickness of the shell and the cross sectional areas of the supporting stiffeners and their locations. In other words, the design variable vector, $\underline{X}_d$ is defined as:

Case I: Un-stiffened shell with linear or parabolic thickness distribution:

$$\underline{X}_d = \{ h_1, h_2 \} \tag{22a}$$

Case II: Un-stiffened shell with general quadratic thickness distribution:

$$\underline{X}_d = \{ h_1, h_3, h_2 \} \tag{22b}$$

Case III: Stiffened shell with uniform wall thickness:

$$\underline{X}_d = \{ h_1, (S_{r,k}, b_{r,k}), k=1, 2, \dots Nr\} \tag{22c}$$

Design variables which are not subject to change in the optimization process are chosen to include: Type of material of construction, total cone height, radii of the clamped and free sections and cross sectional shape of stiffeners. The mass equality constraint is used herein to eliminate any one of the design variables, which results in a more simplified optimization problem having reduced dimensionality. Other side constraints are imposed on the design variables for geometrical, manufacturing or logical reasons. The thickness of the shell wall is usually restricted by the available standard sheet thicknesses. Its lower bounds may also be determined from the consideration of wall instability that might happen by local buckling.

5.2 Optimization Analysis: The present design optimization model belongs to non-linear mathematical programming problems[13-14]. it takes the following form:

$$\text{Minimize } F = - f_{min} \tag{23a}$$

$$\text{subject to} \quad \begin{aligned} & h_L \le h_i \le h_U, \quad i=1, 2, 3 \\ & 0 \le S_{r,k} \le L \\ & b_{r,k} \ge 0, \; k=1,2,\dots,Nr \end{aligned} \tag{23b}$$

where f_{min} is the minimum (fundamental) frequency of the shell structure and h_L and h_U are the lower and upper bounds imposed on the shell wall thickness.

6. Computational Results: The above-simplified model has been applied to arrive at the needed optimal designs of both divergent and convergent conical shells with clamped/free boundary condition. The MATLAB Optimization Toolbox[15] offers routines that implement the interior penalty function method via a built-in function named "fminsearch". Its facilities are invoked for interacting to the routines, which calculates the required numerical values of the original objective function and constraints. Various designs have been obtained for the cases of unstiffened shells with variable thickness distribution as well as stiffened types with uniform shell thickness.

6.1 Definition of a Baseline Design: Before performing the actual optimization analysis, it is useful to define a baseline or guide design with which the subsequent optimum solutions can be compared. Table (1) gives the pertinent data of the selected baseline design, which is described by an un-stiffened metallic conical shell structure having uniform thickness distribution. The free vibration analysis using the method of Rayleigh-Ritz has been tested for calculating the natural frequencies of the baseline design. Results were compared with those calculated by ANSYS- finite element software[16] in order to verify the accuracy of the present mathematical formulation. Three modal analyses with increasingly finer mesh were carried out in ANSYS using 8-node structural shell element, named "SHELL93", with six degrees of freedom at each node.

Table 1: Baseline design data of a metallic conical shell with uniform thickness distribution.

Parameter	Design data	
Cone height, H.	1.3 m	
Shell thickness, h_o.	0.01 m	
Radii at clamped and free sections (R_{1o}, R_{2o}) and apex angle α.	Divergent Case	(0.25, 1.0) m, α=30º
	Convergent Case	(1.0, 0.25) m, α=-30º
Material of construction 1020 corrosion, heat-resistant steel	Mass density	: ρ= 7850 kg/m3
	Young's modulus	: E= 208 Gpa
	Poisson's ratio	: υ=0.3
Total structural mass, M_o	462.38 kg	
Fundamental frequency, f_o	41.7 Hz, for the divergent case.	
	248.0 Hz, for the convergent case.	

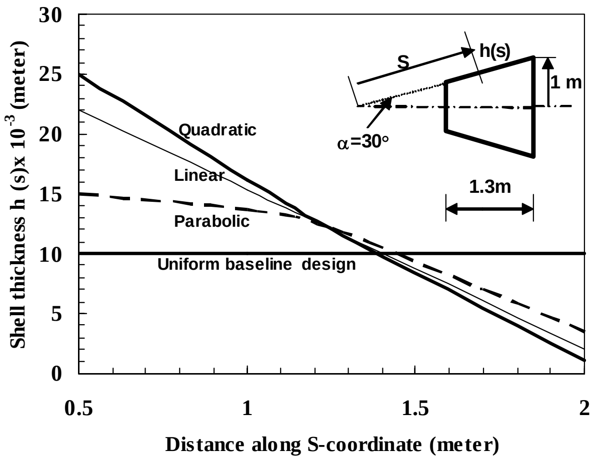
Table 2: Optimal thickness distributions for unstiffened conical shells.

Thickness distribution	Shell configuration	Optimal values at clamped, middle and free sections (h_i, i=1, 3, 2) $x10^{-3}$ m	Fundamental frequency, f_{min} (Hz)	Optimization gain (% increase)
Linear	Divergent	(22.0, 11.98, 2.014)	65.3762	56.78 %
	Convergent	(5.0, 11.235, 17.47)	272.544	9.89 %
Parabolic	Divergent	(15, 12.115, 3.44)	55.1177	32.18 %
	Convergent	(5.0, 10.38, 26.502)	324.13	30.7 %
General Quadratic	Divergent	(25, 12.04, 1.142)	72.562	74.0 %
	Convergent	(7.0, 8.376, 38.485)	288.71	16.415 %

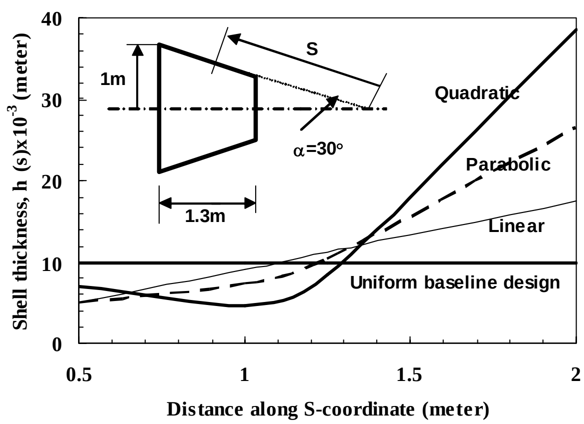
Convergence has been achieved in almost all of the results. The lowest frequency was found by Rayleigh-Ritz method to be 41.7 Hz for the divergent configuration and 248.0 Hz for the convergent configuration, both occurring at a circumferential wave number n=3. The corresponding values computed by ANSYS were 40.45 and 241.0, respectively, occurring at n=3. This proves that the present analytical model can be very efficient in view of eliminating the tedious efforts for preparing finite elements data and saving much of the computational time necessary for solving large systems of equations during the optimization process.

6.2 Optimum patterns with variable thickness: Various optimum designs for unstiffened shells with variable thickness distribution have been obtained. Fig. 2 shows the attained optimum patterns of the shell thickness along the slant length of the conical shell. Numerical results are tabulated in Table 2 for both divergent and convergent configurations. The values of the thickness are given at the clamped, middle and free sections of the shell structure. The shell geometry, as described by the parameters (R_1, R_2, H), material properties and total structural mass are taken the same as those of the baseline design. The thickness at the clamped edge (h_1) can be obtained using the mass equality constraint. It is seen from the results that for the divergent case, decreasing the shell thickness linearly from the clamped edge towards the free edge results in a noticeable increase in the fundamental frequency. On the contrary, for the convergent case, it is recommended to have an increasing rate towards the free end, where the gain in maximizing the fundamental frequency reaches a value of about 19.0%.

It was also found that, for the divergent case, decreasing the thickness parabolically in the direction of the s-coordinate has a much more positive effect on increasing the fundamental frequency than in the case with linear thickness variation. The opposite result is true for the case of convergent configuration, where a sharp reduction in the frequency can be observed for decreasing the thickness towards the free end. For the divergent configuration shown in Fig.2a, the obtained results indicate that optimal patterns with quadratic thickness variation are better than those having either linear or parabolic thickness distribution. The maximum



(a) Divergment configuration



(b) Convergent configuration

Fig. 2: Optimal patterns of thickness distribution for unstiffened conical shells having constant structural mass.

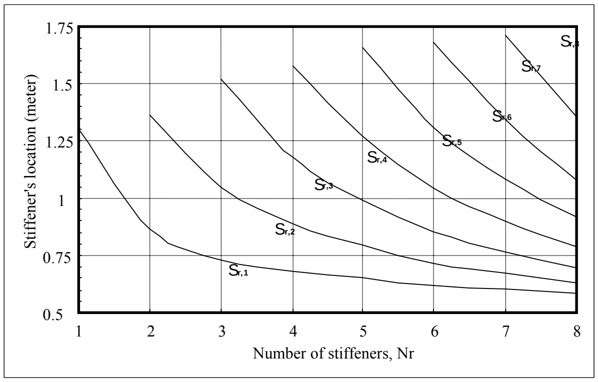
fundamental frequency achieved was computed to be 72.562 HZ, representing an optimization gain of about 74%. It is also observed that the shell thickness at the free end reached its minimum allowable value for the attained optimized designs.

If one were further considering the acceptability of manufacturing cost, it would be recommended to choose optimum patterns having linear thickness distribution, though they have less optimal value of the fundamental frequency. On the contrary, for the convergent configuration the optimization favors smaller thickness at the clamped edge, as indicated in Fig.2b. It is seen that the parabolic distribution excels both of the linear and the general quadratic, where the achieved maximum fundamental frequency reaches a value of 324.13 HZ, corresponding to an optimization gain of about 30.7%. Such an optimum result could violate strength considerations at the clamped end of the shell, where critical shearing stresses in severe loading conditions might cause large damage of the whole shell structure. Therefore, a multi-objective optimization model is preferable for appropriate design of convergent shell structures, considering simultaneous minimization of vibration level and maximization of structural safety.

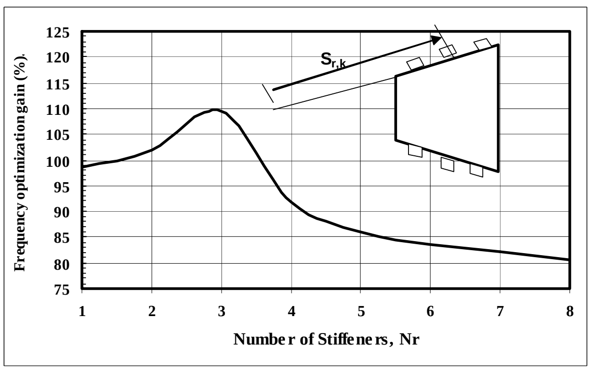
Another noticeable remark, shown in Fig.2b, is that the general quadratic distribution resulted in an odd-shaped optimum pattern having a lower frequency, which might also violate manufacturing and economical requirements.

6.2.1 Effect of the minimum allowable thickness constraint: Computer results have shown that the attained optimum solutions are strongly affected by the lower limiting value assigned to the shell thickness. For divergent configuration, it was found that the thickness at the free end, which has the largest diameter, always sinks towards its lower limiting value, h_L. As this value decreases, the optimal linear pattern was observed to have a sharp decreasing rate towards the free end. The same optimization gain can be achieved for the parabolic distribution, but with a higher value for the minimum allowable thickness at the free end. Therefore, if one considers strength requirements in the formulation of the optimization model, it will be better to select shells with parabolic thickness distribution. On the contrary, it is concluded that good convergent shell designs ought to have thinner thickness at the clamped edge. In all the attained optimal solutions, the minimum thickness constraint was always active, where the thickness at the clamped end reached its lower limiting value, whether the distribution is linear or parabolic. Addition of stress constraints to the optimization model is necessary, especially in the case of convergent shells.

6.3 Optimum Patterns of Stiffened constructions: The proposed optimization model is now utilized to select the optimum design parameters of stiffened metallic conical shells. The stiffening rings are assumed to have solid square cross-section with the same size, b_r and same material type of the shell structure. The number of stiffeners (N_r) is taken as a preassigned parameter ranging from one to eight. The remaining variables are, therefore, the shell thickness h_1, width of the stiffeners b_r and the locations of the individual stiffeners $S_{r,k}$, k=1,2,…N_r. For divergent shells, the maximum achievable fundamental frequency was calculated to be 87.37 HZ, corresponding to three stiffening rings located respectively at 0.731, 1.051 and 1.521 meter, measured from the cone vertex. This indicates that optimization favors stiffeners to be closer to the clamped end having the smaller diameter in

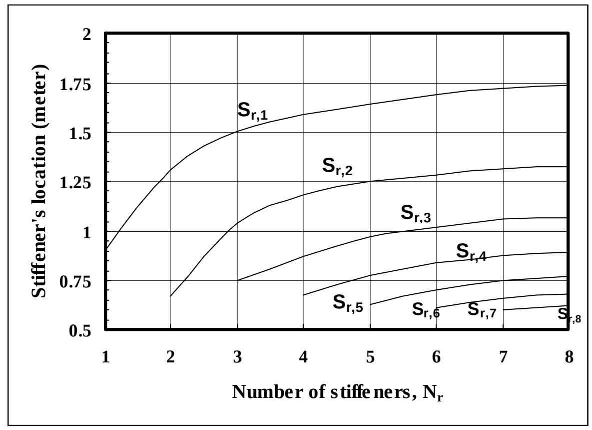


(a) Optimal location of stiffeners



(b) Optimization gain in the fundamental frequency

Fig. 3: Optimal frequency design of stiffened divergent conical shells.



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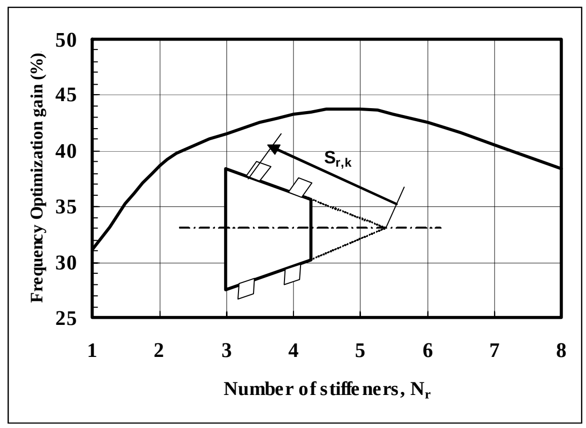

(b) Optimization gain in the fundamental frequency

Fig. 4: Optimal frequency design of stiffened convergent conical shells.

order to increase the frequency. The corresponding optimum values of h_1 and b_r were calculated to be 5.007 mm and 50.9 mm, respectively. The optimal design trend of variation of the stiffener locations with their number is depicted in Fig. 3a. As a general observation, the location of any stiffener is decreasing with increasing the number of stiffening rings, which means that good designs ought to have more stiffeners near the clamped edge of the shell. Such behavior is similar to the case of unstiffened divergent shells with variable thickness distribution, where it was shown that optimization favors higher thickness in the neighborhood of the clamped edge. It becomes now clear that, for divergent con figuration, whether they are stiffened or not, it is recommended to have extra material near the clamped edge with the smallest diameter in order to raise the fundamental frequency while preserving the total structural mass at a constant value. Fig. 3b shows the attained optimization gain for any preassigned value of N_r. It is seen that the peak value equals to about 110% (i.e. f_{min}= 2.1x41.7=87.6 HZ) and occurring at N_r=3, after which the gain decreases rapidly as the number of stiffeners is further increased.

Results for convergent configuration are presented in Fig.4. The maximum attainable optimization gain is seen to be about 43.8% (i.e. f_{min}=1.438x248=356.6 HZ) occurring at N_r=5. The corresponding optimal values of the shell thickness and stiffener width have been computed to be 5.01mm and 39.2 mm, respectively. It is seen that good designs shall have more stiffeners concentrated near the free end with the smallest diameter, which is opposite to the case of divergent shells.

7. Conclusions and Future Aspects: In view of the practical use of shell structures in several engineering applications, a model for optimizing dynamic performance of typical conical shells has been developed and applied to structural types with clamped-free boundary conditions. Both stiffened and unstiffened metallic constructions having either divergent or convergent configuration have been examined. The objective function is measured by maximization of the fundamental frequency under constant structural mass with the design variables including the shell thickness, size and location of stiffeners. Side constraints imposed on the values of the design variables are also considered to avoid having negative values or odd-shaped configurations in the resulting optimum solutions. Structural analysis is based on Donnell-Mushtari shell theory and the method of Rayleigh-Ritz has been applied to calculate the natural vibration characteristics. Results have shown that the proposed optimization model succeeds in arriving at the optimum values of the selected design variables corresponding to any desired case. Conspicuous optimum trends have been obtained for good designs of the different conical shell structural types.

It has been shown that optimal patterns, whether stiffened or not, shall have material distribution with more concentration in the neighborhood of the edge having the smallest diameter, whatever its support condition. This means that maximization of the fundamental frequency alone under the mass constraint does not ensure the required safety of the whole shell structure. Therefore, additional design objectives and constraints regarding important strength requirements should be considered in future optimization analysis. It has been also demonstrated that the stiffened construction is much better than the unstiffened one in achieving the highest possible frequency without the penalty of increasing structural mass.

There are still many factors and different approaches that can be considered in future optimization of shell structures. For example, another direction is to consider the dual problem of minimizing the structural weight while preserving the important frequencies at their desired values. In this aspect, the optimality criterion method, which has wide applications for obtaining minimum weight design subject to frequency constraints, can be applied. Research work can also be extended to consider a more comprehensive optimization formulation involving many design parameters and applying multi-criteria optimization techniques[17] in order to simultaneously minimize several design objectives such as, noise, vibration, structural weight, buckling, fatigue and cost.

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