

Quadratic Error Optimization Algorithm Applied to 3D Space Distributed Array Sensors

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Abstract: 3/4Beamforming sensors hold key–position in measurement signal conversion and treatment. Indeed, sensors interact directly with environment, and transform signals into an appropriate format. Often environment has several sources, and the generated signals have continuously changing parameters. We must then change the array sensors attitude to cope with environment specifications. In this article we present the quadratic error optimization algorithm and its implementation to adapt and control 3D space distributed array sensors for space filtering applications. We will present results for two array architectures linear and spherical.

Key words: 3/4Sensors arrays, acquisition, Beam forming, adaptive array sensors.

INTRODUCTION

In measurements we are fairly often confronted to unknown or continuously changing environment parameters. Moreover in many applications the main problem consists of adapting array sensors parameters to current environment specifications. Indeed, sensors are always affected by multiple sources signals. In this article we will show how to implement the minimum square error algorithm to adjust 3D space distributed array sensors response. Beamforming is obtained by changing the complex ponderations associated with each sensor^[1].

MATERIAL AND METHODS

1. Method Scope: The technique based on the mean square error minimisation was first introduced by WIDROW for uniformly spaced linear array antennas^[2,3]. We have improved this method, and we have adapted it to space distributed sensors^[1].

2. Algorithm Formulation: First An error $e(t)$ is estimated by calculating the difference between the reference signal $d(t)$ and the array response $y(t)$ given by equation (1) as shown in Fig. 1^[2,3]. In our case the reference signal $d(t)$ is highly correlated with the valuable information signal (the signal to be measured). The error is expressed by equation (2).

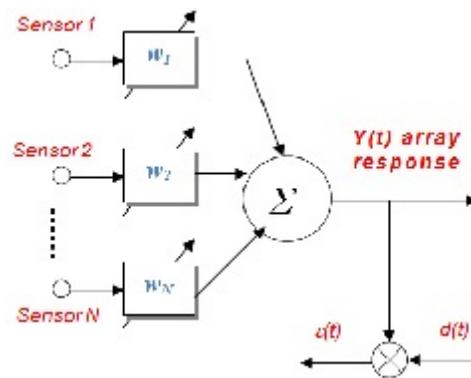


Fig. 1: Error estimation.

$$y(t) = W^H \cdot X(t) \tag{1}$$

$$\varepsilon(t) = d(t) - W^H \cdot X(t) \tag{2}$$

where $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}$ are the sensors responses.

and $W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{bmatrix}$ = complex sensors ponderations.

The equation (3) calculates equation (2) square. We will then evaluate the quadratic error mean value by equation (4).

$$\varepsilon(t)^2 = d(t)^2 - 2d(t)W^H X(t) + W^H X(t)X(t)^H W \tag{3}$$

$$E\{\varepsilon(t)^2\} = \overline{d(t)^2} - 2\text{Re}(W^H r_{xd}) + W^H R_{xx} W \tag{4}$$

where

$$r_{xd} = \begin{bmatrix} \overline{x_1(t)d(t)} \\ \overline{x_2(t)d(t)} \\ \vdots \\ \overline{x_N(t)d(t)} \end{bmatrix}$$

and

$$R_{xx} = \begin{bmatrix} \overline{x_1(t)x_1(t)} & \overline{x_1(t)x_2(t)} & \dots & \overline{x_1(t)x_N(t)} \\ \overline{x_2(t)x_1(t)} & \overline{x_2(t)x_2(t)} & \dots & \overline{x_2(t)x_N(t)} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{x_N(t)x_1(t)} & \overline{x_N(t)x_2(t)} & \dots & \overline{x_N(t)x_N(t)} \end{bmatrix}$$

The method consists of minimizing the mean square error given by equation (4). This minimization will be done by an appropriate selection of the complex ponderations values « w_i ». However, the equation (4) has a quadratic form; its extremum is automatically a minimum. The extremum will be estimated by resolving the expression (5).

$$\Delta E\{\varepsilon(t)^2\} = 0 \tag{5}$$

We have solved the equation (5), and fined that the optimal complex ponderations are expressed by equation (6). For the implementation we have used the gradient steepest descent optimization algorithm^[4].

$$W_{opt} = R_{xx}^{-1} r_{xd} \tag{6}$$

3. Array Elements Reduction: We haven't used all the array elements. Indeed, in our case sensors can be aleatory distributed in space. It is useless to consider the response of sensor pointing in a wrong direction. The different sensors don't receive signals with the same power and we suppose inexistent sensors having response power under a threshold. This is done by associating zero as complex ponderation value for those sensors as shown in Fig. 2. With this solution we have simplified the array architecture, and reduced iterations made by the minimization algorithm.

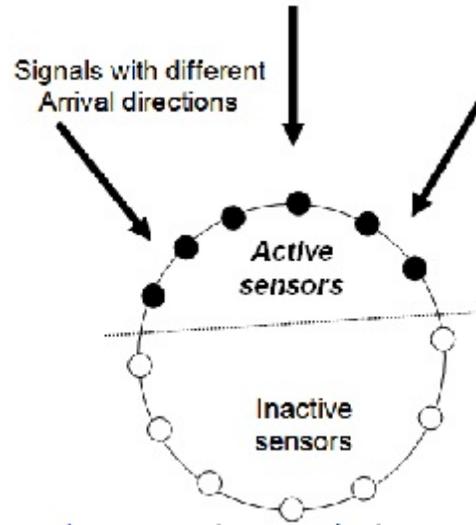


Fig. 2: Sensor elements reduction.

4. Simulations and Results: This approach based on the minimum square error optimization was implemented and we have done simulations for different array sensors configurations.

We start by presenting in Fig.3 the simulation result for 7 linear array sensors. The interference sources were generated with -10° , 20° , and 30° as arrival directions. The useful signal has 10° arrival direction.

As shown in Fig.3 response between -40dB to -90dB was done in the noise directions. The main lobe was deviated to desired direction.

In Fig.4 we present spherical beam pattern of an array with 36 sensors. The useful signal arrival direction is imposed to $[0^\circ 30^\circ]$ and $[-100^\circ 20^\circ; -30^\circ 50^\circ; 0^\circ -75^\circ; 20^\circ -45^\circ; 45^\circ 150^\circ]$ the directions of 5 undesired signals. Fig.5 shows that we have systematic interference rejection of undesired signals with main lobe deviated to valuable signal direction.

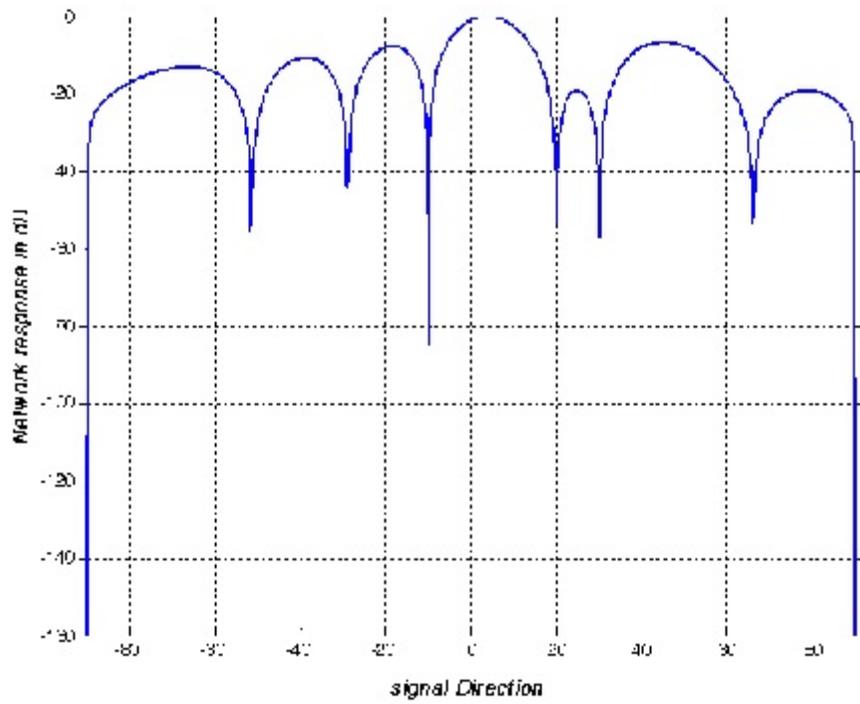


Fig. 3: beam pattern 7 elements linear array.

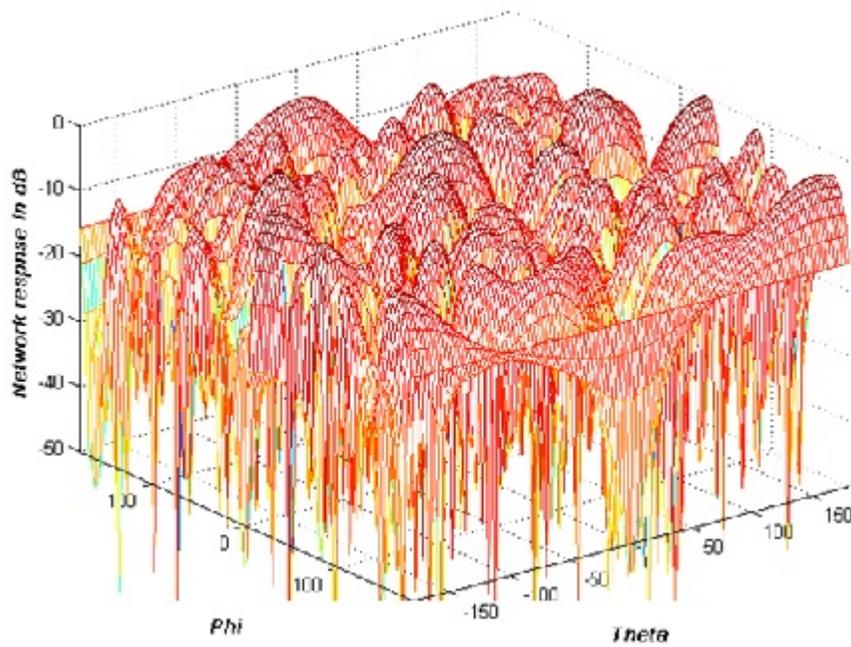
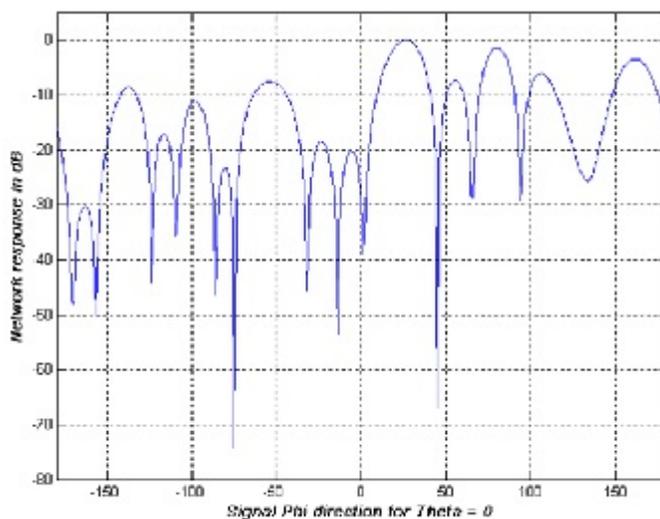
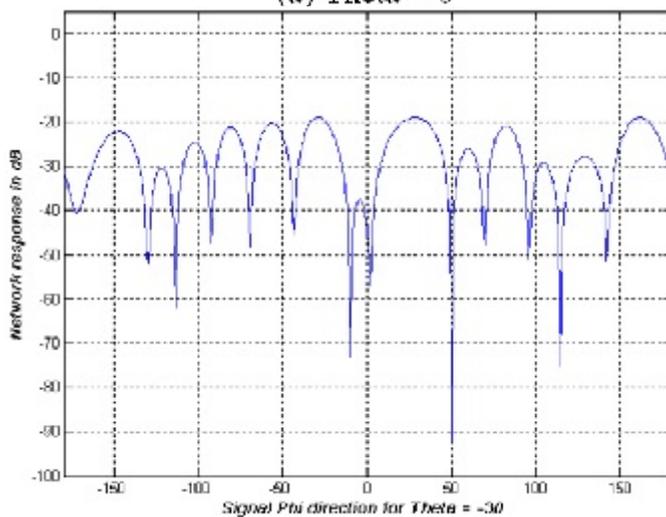


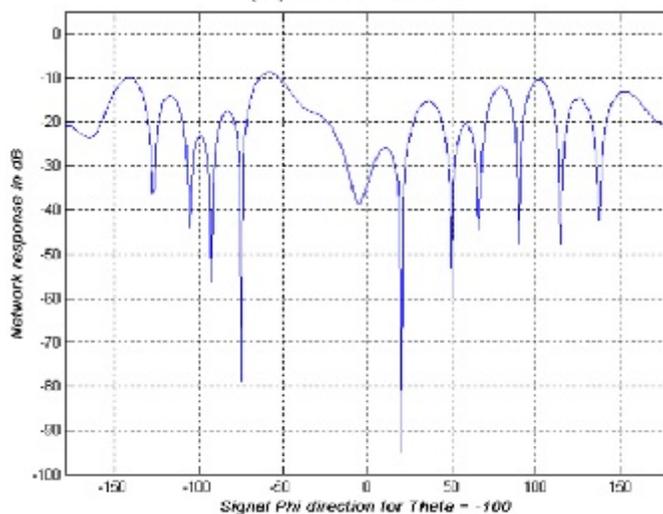
Fig. 4: beam pattern 36 elements spherical array sensors.



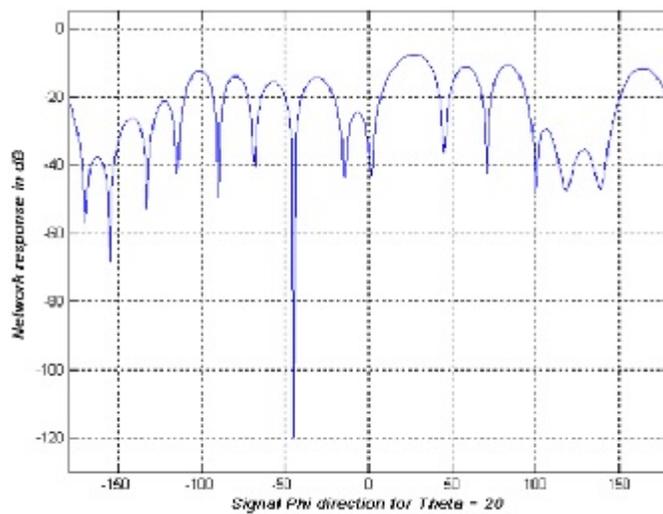
(a) Theta = 0



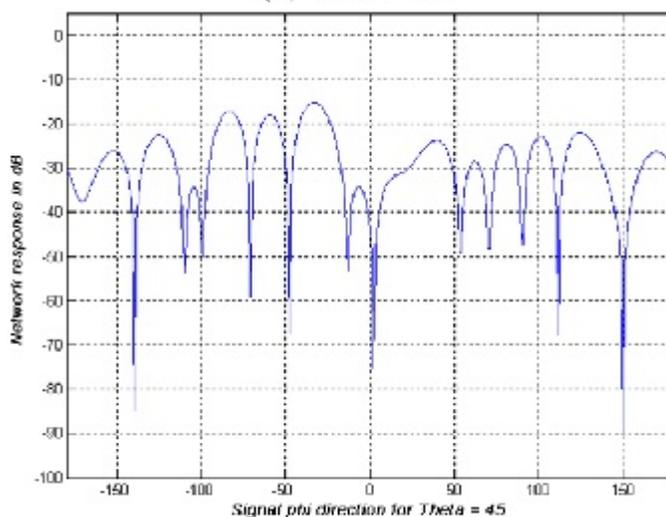
(b) Theta = -30



(c) Theta = -100



(d) Theta = 20



(e) Theta = 45

Fig. 5: 36 elements spherical array sensors response.

IV. Conclusion: In this article we have introduced an approach based on the quadratic error algorithm to adapt and control 3D space distributed array sensors. We have done several simulations with many array architectures. This method produced directivity diagrams with variable shapes and directions. It also copes with spatial restrictions imposed by environment.

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