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Estimation of the Unknown Parameters of the Generalized Frechet Distribution

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Abstract: The extreme value distribution is becoming increasingly important in engineering statistics as a suitable model to represent phenomena with usually large maximum observations. In engineering circles, this distribution is often called the Frechet model. It is one of the pioneers of extreme value statistics. The Frechet (extreme value type II) distribution is one of the probability distributions used to model extreme events. . In the present study the maximum likelihood estimation of the parameters of generalized Frechet (GF) distribution are derived. Asymptotic variance covariance matrix are derived and computed numerically.

Key words: Fisher Information, generalized Frechet distribution, maximum likelihood estimator, sampling distribution, Pearson system.

INTRODUCTION

In recent years, several standard life time distributions have been generalized via exponentiation. Examples of such exponentiated distributions are the exponentiated Weibull family, the exponentiated exponential, the exponentiated Rayleigh, the exponentiated (generalized) Frechet and the exponentiated Pareto family of distributions.

Amongst the authors who have considered the exponentiated distributions are Mudholkar & Hutson[5,1,7,6,3,4]. A common feature in families of exponentiated distributions is that the distribution function may be written as $F(x) = \left[G(x)\right]^{\alpha}$ where $G(.)$, is the distribution function of a corresponding non generalized distribution and α > 0 denotes the generalized parameter. The generalized Frechet distribution is obtained by generalization of the Frechet distribution.

Recently, a new three parameter distribution, named as Exponentiated Frechet (EF) distribution has been introduced by Nadarajah & Kotz[6] as a generalization of the standard Frechet distribution. There are over fifty applications ranging from accelerated life testing through to earthquakes, floods, horseracing, rain fall, queues in supermarkets, sea currents, wind speeds and track race records, see Kotz and Nadarajah[2].

The distribution function is

$$F(x;\alpha,\lambda,\sigma) = 1 - \left[1 - \exp\left\{-\left(\frac{\sigma}{x}\right)^{\lambda}\right\}\right]^{\alpha}, \qquad x > 0 \tag{1}$$

where $\alpha > 0$, $\lambda > 0$ the shape parameters and $\sigma > 0$ the scale parameter.
The probability density function is

$$f(x,\alpha,\lambda,\sigma) = \alpha\lambda\sigma^{\lambda}\left[1 - \exp\left\{-\left(\frac{\sigma}{x}\right)^{\lambda}\right\}\right]^{\alpha-1} x^{-(\lambda+1)} \exp\left\{-\left(\frac{\sigma}{x}\right)^{\lambda}\right\}, \; x > 0 \tag{2}$$

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Figure. (1) shows the probability density functions of the generalized Frechet distribution for shape parameter α = 0.25, 0.5, 1, 2 and 2.5 ; σ = 1, λ = 1

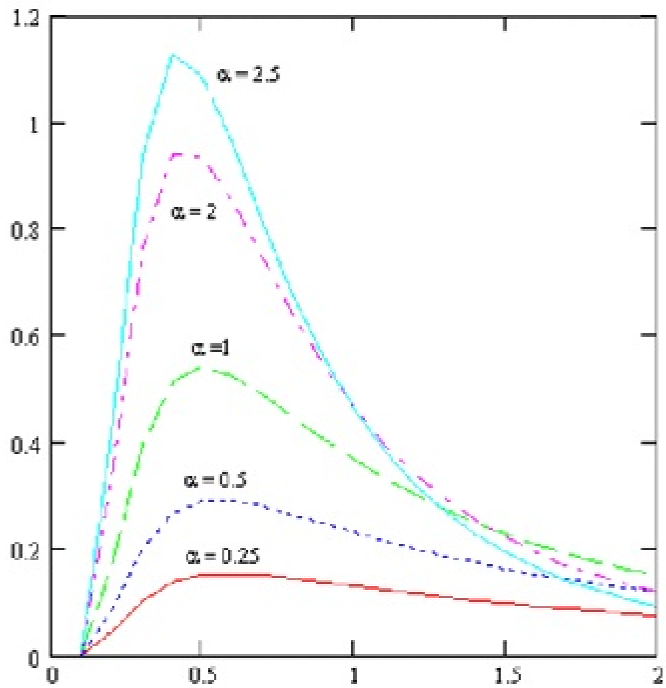

Fig. 1: The density functions of the generalized frechet distribution for different values of shape parameter.

Its survival function is

$$S(x;\alpha,\lambda,\sigma)=\left[1-\exp\left\{-\left(\frac{\sigma}{x}\right)^{\lambda}\right\}\right]^{\alpha} \quad 3)$$

The hazard rate function is an important quantity characterizing the life phenomena of the distribution. Its hazard function is

$$h(x,\alpha,\lambda,\sigma)=\frac{\alpha\lambda\sigma^{\lambda}x^{-(\lambda+1)}\exp\left\{-\left(\frac{\sigma}{x}\right)^{\lambda}\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x}\right)^{\lambda}\right\}\right]} \quad (4)$$

Figure (2) illustrates some of the possible shapes of the hazard functions for selected values of shape parameters α = 0.25, 0.5, 1, 2 and 2.5 ; σ = 1, λ = 1

The main aim of this paper is to consider the analysis of the unknown parameters for the generalized Frechet distribution. We obtain the maximum likelihood estimators (MLE) of unknown parameters of the GF distribution. It is observed that the MLE for the unknown parameters can not be obtained in explicit from. Finally, we obtain the Fisher information matrix.

The article is arranged as follows. In section 2, the parameter estimation method is presented. In section 3, the sampling distribution for the unknown parameters is obtained by using Monte Carlo simulation. Section 4, contains the asymptotic variance covariance matrix.

2. Maximum Likelihood Estimators: In this section, the maximum likelihood estimators of GF (α, λ, σ) are considered in case of λ known.

If x_1, x_2,....x_n is a random sample from the Exponentiated Frechet distribution GF (α, λ, σ), then the likelihood function corresponding to this sample is given by

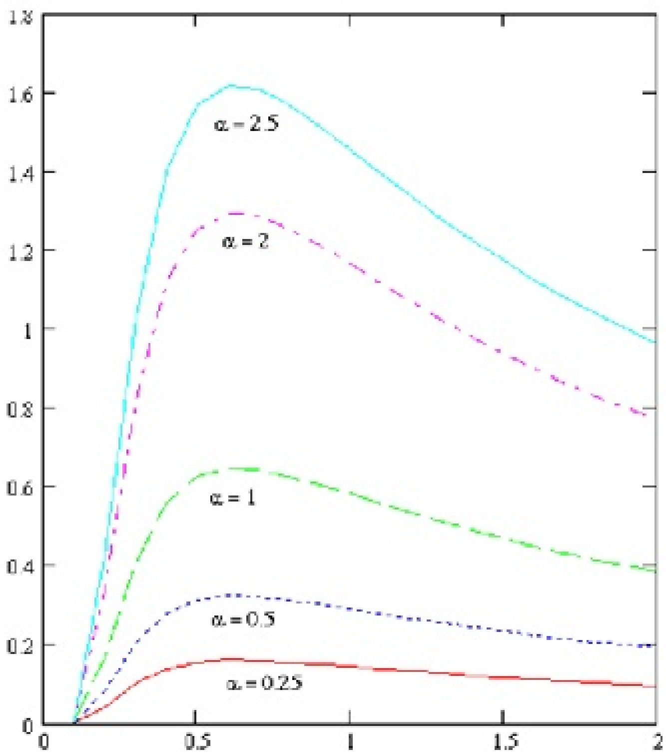

Fig. 2: The hazard rate functions of the generalized frechet distribution for selected values of shape parameter.

$$\prod_{i=1}^{n} f(x_i,\alpha,\lambda,\sigma)=\prod_{i=1}^{n}\left[\alpha\lambda\sigma^{\lambda}\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)^{\lambda}\right\}\right]^{\alpha-1} x_i^{-(\lambda+1)}\exp\left\{-\left(\frac{\sigma}{x_i}\right)^{\lambda}\right\}\right] \quad (5)$$

The log likelihood function is

$$\ln L = n\ln(\alpha\lambda)+n\lambda\ln\sigma+(\alpha-1)\sum_{i=1}^{n}\ln\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)^{\lambda}\right\}\right]-\left[(1+\lambda)\sum_{i=1}^{n}\ln x_i+\sum_{i=1}^{n}\left(\frac{\sigma}{x_i}\right)^{\lambda}\right] \quad (6)$$

When the parameter λ is known, we differentiate the log likelihood function with respect to α and σ respectively to obtain the likelihood equations

Therefore

$$\frac{\partial \ln L}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\ln\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]=0 \quad (7)$$

And

$$\frac{\partial \ln L}{\partial \sigma}=\frac{n}{\sigma}+(\alpha-1)\sum_{i=1}^{n}\frac{\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\left(\frac{1}{x_i}\right)}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]}-\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)=0 \tag{8}$$

Then, the maximum likelihood estimator of α as a function of σ is obtained from (7) as

$$\hat{\alpha}(\sigma)=\frac{-n}{\sum_{i=1}^{n}\ln\left[1-\exp\left[-\left(\frac{\sigma}{x_i}\right)\right]\right]} \tag{9}$$

Substitute from (9) in (8) and solve for σ, we obtain

$$\frac{n}{\hat{\sigma}}+(\hat{\alpha}(\sigma)-1)\sum_{i=1}^{n}\frac{\exp\left\{-\left(\frac{\hat{\sigma}}{x_i}\right)\right\}\left(\frac{1}{x_i}\right)}{\left[1-\exp\left\{-\left(\frac{\hat{\sigma}}{x_i}\right)\right\}\right]}-\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)=0 \tag{10}$$

We apply iterative procedure to find the solution of (10), once we obtain $\hat{\sigma}$ The maximum likelihood estimators of a can be obtained from (9).

3. Sampling Distributions of the Maximum Likelihood Estimators: In this section, the sampling distributions of the maximum likelihood estimators for the generalized Frechet distribution are derived using Pearson system. The Pearson system embeds seven basic types of distribution together in a single parametric framework. The selection approach is based on computing a certain quantity, *K*, which is a function of the first four central moments, that is:

$$K=\frac{\beta_1\cdot(\beta_2+3)^2}{4\cdot(4\beta_2-3)\cdot(2\beta_2-3\beta_1-6)}$$

Where β_1 and β_2 the measures of skewness and kurtosis respectively. This value of *K* differs among the types of Person curves.

If $K < 0$, we fit type I of Persons curve , while type II is fitted if $K = 0$ and $\beta_2 > 3$ .If $K = \infty$, we obtain type III . We get type IV if $0 < K < 1$. When $K=1$, type V is obtained. If $K > 1$, get type VI. Finally, if $K = 0$ and $\beta_2 < 3$ type VII is obtained

The implementation of the Pearson system approach could be summarized in the following steps:

Step (1): A random sample $x_1, x_2,....x_n$ from the Exponentiated Frechet was generated. Firstly a random sample $U_{(1)}, U_{(2)},..., U_{(n)}$ of n order statistics from a uniform (0,1) distribution was generated, then the i[th] order statistics from the EF(α,l,σ) with α=1 ,λ=1and s=1 will be obtained as follows

$$X_{(i)}=\left[\frac{\sigma}{\left[-\ln\left(1-\left(1-U_{(i)}\right)^{\frac{1}{\alpha}}\right)\right]^{\frac{1}{\lambda}}}\right] \quad , i = 1,2,.....,n$$

Step (2): This random sample was used to estimate the unknown parameters by method of maximum likelihood mentioned in section 2.

Step (3): The mean, standard deviation, skewness, kurtosis and Pearson's coefficient are calculated for each estimators of the unknown parameters and sample size.

As a result of computer simulation; three Pearson distributions were fitted to the maximum likelihood parameters estimators which are Pearson type I, IV and VI distributions.

Pearson type I distribution has the probability density function:

$$P(x) = k(x-a_1)^{m_1} \times (a_2-x)^{m_2} \quad , a_1 < x < a_2$$

Where α_1 and a_2 are the roots of the equation $c_o + c_1x + c_2x^2 = 0$ with $m_1 = \dfrac{a+a_1}{c_2(a_2-a_1)}$

$m_2 = \dfrac{a+a_2}{c_2(a_2-a_1)}$

While Pearson type IV has the density function:

$$p(x) = k\left[c_0 + c_2(x+c_1)^2\right]^{-(2c_2)^{-1}} \exp\left(-\frac{a-c_1}{\sqrt{c_2c_0}} \cdot \tan^{-1}\frac{x+c_1}{\sqrt{c_0/c_2}}\right), -\infty < x < \infty$$

The last one, the density function of Pearson type VI is given by:

$$P(x) = k(x-a_1)^{m_1} \times (x-a_2)^{m_2}, x > a_2$$

The sampling distributions of the maximum likelihood estimators with different sample sizes are listed in Table (1).

4. Asymptotic Variance Covariance Matrix: Fisher Information matrix $I(\alpha,\sigma)$ can be written as follows:

$$I(\alpha,\sigma) = -\frac{1}{n}\begin{pmatrix} E\left(\dfrac{\partial^2 L}{\partial\alpha^2}\right) & E\left(\dfrac{\partial^2 L}{\partial\alpha\cdot\partial\sigma}\right) \\ E\left(\dfrac{\partial^2 L}{\partial\sigma\cdot\partial\alpha}\right) & E\left(\dfrac{\partial^2 L}{\partial\sigma^2}\right) \end{pmatrix} \quad (11)$$

The elements of the Fisher Information matrix can be written as

$$E\left(\frac{\partial^2 \ln L}{\partial\alpha^2}\right) = -\frac{n}{\alpha^2} \quad (12)$$

Table 1: Sampling Distribution of Parameter Estimators From Generalized Frechet

Sample Size n	parameters	Mean	Var.	Mean Square error	skewness	kurtosis	Person's coefficient	Type
10	σ	4.144	48.384	6.957	10.734	143.186	0.406	IV
	α	4.658	8.065	2.84	4.992	70.898	0.201	IV
15	σ	3.151	11.035	3.324	22.674	634.333	0.76	IV
	α	1.581	0.097	0.312	0.775	4.465	1.201	VI
20	σ	2.868	2.201	1.484	15.868	554.848	0.527	IV
	α	4.128	2.126	1.458	1.75	9.138	0.273	IV
25	σ	2.753	0.824	0.908	1.406	7.016	0.369	IV
	α	4.052	1.503	1.226	1.503	7.363	0.362	IV
30	σ	2.683	0.625	0.791	1.176	5.492	0.768	IV
	α	3.997	1.164	1.079	1.274	6.447	0.406	IV
35	σ	2.615	0.48	0.693	1.004	4.789	1.665	VI
	α	3.969	0.97	0.985	1.048	4.957	1.28	VI
40	σ	2.607	0.406	0.637	0.922	4.397	30.877	VI
	α	3.921	0.774	0.88	0.92	4.588	2.074	VI
45	σ	2.584	0.355	0.596	1.053	6.065	0.342	IV
	α	3.912	0.702	0.838	1.006	5.049	0.877	IV
50	σ	2.549	0.297	0.545	0.798	4.008	-1.989	I
	α	3.891	0.625	0.791	0.866	4.491	2.114	VI
55	σ	2.514	0.257	0.507	0.833	4.567	1.23	VI
	α	3.865	0.558	0.747	0.848	4.054	-1.831	I
60	σ	2.516	0.245	0.495	0.877	4.539	1.839	VI
	α	3.857	0.499	0.707	0.832	4.534	1.364	VI
65	σ	2.511	0.234	0.484	0.855	4.481	2.019	VI
	α	3.855	0.442	0.665	0.817	4.953	0.528	IV
70	σ	2.519	0.209	0.457	0.748	4.187	5.404	VI
	α	3.85	0.41	0.641	0.679	3.86	-2.026	I
75	σ	2.493	0.187	0.433	0.676	3.754	-1.234	I
	α	3.838	0.373	0.611	0.613	3.811	-2.676	I
80	σ	2.49	0.183	0.428	0.749	4.383	1.353	VI
	α	3.837	0.377	0.614	0.77	4.353	1.824	VI
90	σ	2.479	0.154	0.393	0.638	3.668	-1.05	I
	α	3.828	0.316	0.563	0.575	3.457	-0.682	I
100	σ	2.46	0.131	0.361	0.502	3.333	-0.58	I
	α	3.824	0.281	0.531	0.524	3.423	-0.696	I

$$E\left(\frac{\partial^2 \ln L}{\partial \alpha \partial \sigma}\right) = E\left(\sum_{i=1}^{n} \frac{\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\left(\frac{1}{x_i}\right)}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]}\right) \tag{13}$$

And

$$E\left(\frac{\partial^2 \ln L}{\partial \sigma^2}\right) = E\left(\frac{-n}{\sigma^2} - (\alpha - 1)\sum_{i=1}^{n} \frac{\left(\frac{1}{x_i}\right)^2 \exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^2}\right) \tag{14}$$

to compute (13) and (14) we obtain explicit expression of expectation of the forms.

$$E\left(\frac{\left(\frac{1}{x_i}\right)\cdot\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]}\right) \text{ and } E\left(\frac{\left(\frac{1}{x_i}\right)^2\cdot\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^2}\right) \text{ for i=1,2,………,n}$$

Since the density of the i^{th} order statistic from random sample of size n following the EF(α,1,σ) distribution

$$f_{x_i}(x)=\frac{n!}{(i-1)!(n-i)!}\alpha\sigma x_i^{-2}\left[\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]\left[1-\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^{\alpha}\right]^{i-1}\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^{\alpha(n-i+1)-1}$$

Then

$$E\left(\frac{\left(\frac{1}{x_i}\right)\cdot\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]}\right)=\int_0^{\infty}\frac{\left(\frac{1}{x_i}\right)\cdot\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]}\cdot f_{x_i}(x)\,dx$$

$$=\frac{n!\cdot\alpha\cdot\sigma}{(i-1)!(n-i)!}\cdot\int_0^{\infty}x_i^{-3}\cdot\left[\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^2\cdot\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^{\alpha(n-i+1)-2}\cdot\left[1-\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^{\alpha}\right]^{i-1}dx$$

$$I_1=\int_0^{\infty}x_i^{-3}\cdot\left[\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^2\cdot\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^{\alpha(n-i+1)-2}\cdot\left[1-\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^{\alpha}\right]^{i-1}dx$$

If put $y=\exp\left\{-\left(\frac{\sigma}{x}\right)\right\}$

$$x=\sigma\cdot\left(\log\left(\frac{1}{y}\right)\right)^{-1};.dx=\sigma\cdot\left(\log\left(\frac{1}{y}\right)\right)^{-2}\cdot y^{-1}\cdot dy; x=0\Leftrightarrow y=0.and.x=\infty\Leftrightarrow y=1$$

$$I_1=\int_0^1\left(\sigma\left(\log\left(\frac{1}{y}\right)\right)^{-1}\right)^{-3}y^2\cdot[1-y]^{\alpha(n-i+1)-2}\cdot\left[1-[1-y]^{\alpha}\right]^{i-1}\left(\sigma\cdot\left(\log\left(\frac{1}{y}\right)^{-2}\cdot y^{-1}\right)\cdot dy\right)$$

$$=\frac{1}{\sigma^2}\int_0^1\left(\log\left(\frac{1}{y}\right)\right)\cdot y\cdot[1-y]^{\alpha(n-i+1)-2}\cdot\left[1-[1-y]^{\alpha}\right]^{i-1}\cdot\cdot dy$$

$$=\frac{1}{\sigma^2}\cdot\sum_{K=0}^{i-1}\binom{i-1}{k}\cdot(-1)^{i-(k+1)}\cdot\int_0^1\left(\log\left(\frac{1}{y}\right)\right)\cdot y\cdot[1-y]^{\alpha(n-i+1)-2}\cdot[1-y]^{\alpha(i-k-1)}dy$$

$$=\frac{1}{\sigma^2}\cdot\sum_{K=0}^{i-1}\binom{i-1}{k}\cdot(-1)^{i-(k+1)}\cdot\int_0^1\left(\log\left(\frac{1}{y}\right)\right)\cdot y\cdot[1-y]^{\alpha(n-k)-2}\cdot dy$$

$$=\frac{1}{\sigma^2}\cdot\sum_{K=0}^{i-1}\binom{i-1}{k}\cdot(-1)^{i-(k+1)}\cdot\sum_{j=0}^{\alpha(n-k)-2}\left[\binom{\alpha(n-k)-2}{j}\cdot(-1)^{\alpha(n-k)-2-j}\right]\cdot\int_0^1\left(\log\left(\frac{1}{y}\right)\right)\cdot y^{\alpha(n-k)-(j+1)}\cdot dy$$

Thus we can use the integral

$$\int_0^1\left(\log\left(\frac{1}{x}\right)\right)^{\mu-1}\cdot x^{\nu-1}\cdot dx=\frac{1}{\nu^{\mu}}\cdot\Gamma(\mu)...[\text{Re}.\mu.>0..,\text{Re}.\nu.>0] \tag{15}$$

When $\mu=2$, $\nu=\alpha$ (n-k)-j and x = y Thus

$$\int_0^1\left(\log\left(\frac{1}{y}\right)\right)\cdot y^{(\alpha(n-k)-j)-1}\cdot dy=\frac{1}{(\alpha(n-k)-j)^2}\cdot\Gamma(2)=\frac{1}{(\alpha(n-k)-j)^2}$$

Therefore

$$E\left(\frac{\left(\frac{1}{x_i}\right)\cdot\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]}\right)$$

$$=\frac{n!\cdot\alpha\cdot\sigma^{-1}}{(n-i)!\cdot(i-1)!}\cdot\sum_{K=0}^{i-1}\cdot\left[\binom{i-1}{k}\cdot(-1)^{i-(k+1)}\right]\sum_{j=0}^{\alpha[n-k]-2}\left[\binom{\alpha(n-k)-2}{j}\cdot(-1)^{\alpha(n-k)-(j+2)}\right]\cdot\left[\frac{1}{(\alpha(n-k)-j)^2}\right]$$

Similarly to compute

$$E\left(\frac{\left(\frac{1}{x_i}\right)^2\cdot\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^2}\right)=\int_0^\infty\frac{\left(\frac{1}{x_i}\right)^2\cdot\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^2}\cdot f_{xi}(x)\,dx$$

$$=\frac{n!\cdot\alpha\cdot\sigma}{(i-1)!\,(n-i)!}\cdot\int_0^\infty x_i^{-4}\cdot\left[\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^2\cdot\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^{\alpha(n-i+1)-3}\cdot\left[1-\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^{\alpha}\right]^{i-1}dx$$

In the same way we use $y=\exp\left\{-\left(\frac{\sigma}{x}\right)\right\}$ the above equation reduce to

$$=\frac{n!\cdot\sigma^{-2}\cdot\alpha}{(n-i)!\cdot(i-1)!}\cdot\sum_{K=0}^{i-1}\binom{i-1}{k}\cdot(-1)^{i-(k+1)}\cdot\sum_{j=0}^{\alpha(n-k)-3}\left[\binom{\alpha(n-k)-3}{j}\cdot(-1)^{\alpha(n-k)-(j+3)}\right]\cdot$$

$$\int_0^1\left(\log\left(\frac{1}{y}\right)\right)^2\cdot y^{((\alpha(n-k)-(j+1))-1)}\cdot dy$$

Then by using (15) when $\mu = 3$, $v = \alpha$ (n-k) - (j+1) and x = y (i.e) $E\left(\frac{\left(\frac{1}{x_i}\right)^2 \cdot \exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}}{\left[1-\exp\left\{-\left(\frac{\sigma}{x_i}\right)\right\}\right]^2}\right)$

$$= \frac{n!\,\alpha\cdot\sigma^{-2}}{(n-i)!\,(i-1)!}\cdot\sum_{K=0}^{i-1}\cdot\left[\binom{i-1}{k}\cdot(-1)^{i-(k+1)}\right]\sum_{j=0}^{\alpha(n-k)-3}\left[\binom{\alpha(n-k)-3}{j}\cdot(-1)^{\alpha(n-k)-3-j}\right]\cdot\left[\frac{1}{\left(\alpha(n-k)-(j+1)\right)^3}\right]$$

Now, we compute the elements of Fisher information matrix for the generalized Frechet distribution numerically. For different sample size n= 10, 15, 20, 25, 30, 35, 40, 45, and 50. Compute $I(\lambda, \sigma)$ which defined in (11).

We compute $\frac{1}{n}E\left(\frac{\partial^2 L}{\partial\alpha^2}\right)$. $\frac{1}{n}E\left(\frac{\partial^2 L}{\partial\alpha.\partial\sigma}\right)$ and $\frac{1}{n}E\left(\frac{\partial^2 L}{\partial\sigma^2}\right)$ corresponding to MLE$_s$ of the parameters (α,σ) of the generalized Frechet distribution in each sample size. The results are listed at Table (2).

Table 2: Elements of the approximate Fisher information matrices of EF(α,1,s)

Sample Size n	MSEs of Parameter	Var(α,α)	Cov (α,s)	Var (s,s)
10	$\hat{\alpha}$ =4.658 $\hat{\sigma}$ =4.144	0.046	-0.281	0.306
15	$\hat{\alpha}$ =1.581 $\hat{\sigma}$ =3.151	0.4	-0.125	0.124
20	$\hat{\alpha}$ =4.128 $\hat{\sigma}$ =2.868	0.059	-0.36	0.514
25	$\hat{\alpha}$ =4.052 $\hat{\sigma}$ =2.753	0.061	-0.368	0.54
30	$\hat{\alpha}$ =3.997 $\hat{\sigma}$ =2.683	0.063	-0.372	0.555
35	$\hat{\alpha}$ =3.969 $\hat{\sigma}$ =2.615	0.063	-0.353	0.574
40	$\hat{\alpha}$ =3.921 $\hat{\sigma}$ =2.607	0.065	$02.92\text{x}10^{-5}$	0.15
45	$\hat{\alpha}$ =3.912 $\hat{\sigma}$ =2.584	0.065	$8.33\text{x}10^{-3}$	0.25

Table 2: Continue

50	$\hat{\alpha}$ =3.891 $\hat{\sigma}$ =2.549	0.065	3.35	0.98

REFERENCES

1. Gupta, R.D. and D. Kundu, 2001. Exponentiated exponential family: an lternative to gamma and Weibull distributions. Biometrical J., 43: 117- 130.
2. Kotz, S. and S. Nadarajah, 2000. Extreme Value Distributions: Theory and Applications. London, Imperial College Press.
3. Kundu, D. and R.D. Gupta, 2007. A convenient way of generating gamma random variables using generalized exponential distribution. Comput. Statist. & Data Anal., 51(6): 2796-2802.
4. Kundu, D. and R.D. Gupta, 2008. Generalized exponential distribution: Bayesian estimation, Comput. Statist.& Data Anal., 52(4): 1873-1883.
5. Mudholkar, G.S. and A.D. Hutson, 1996. Exponentiated Weibull family: Some properties and flood data application. Comm. Statist. Theeor. Methods, 25: 3050-3083.
6. Nadarajah, S. and S. Kotz, 2003. The Exponentiated Frechet Distribution. Interstat Electronic Journal, http://interstat.statjournals.net/YEAR/2003/articles/0312001.pdf
7. Surles, J.G. and W.J. Padgett, 2001. Inference for reliability and stress-strength for a scaled Burr type X distribution, Lifetime Data Analysis, 7: 187-200.