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Long Memory Forecasting of Stock Price Index Using a Fractionally Differenced Arma Model

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Abstract: In this paper we investigated the long memory of Stock Price Index (TSIP) and fitted a fractionally differenced ARMA Model using 970 daily data during 26th March 2003 to 8th July 2007 from Tehran stock Exchange. Furthermore, we compared the forecasting outcome of ARFIMA and ARIMA models. The results show that the series is long memory and therefore it can become stationary with fractional differencing. After processing fractional differencing and determining the number of lags of the autoregressive and moving average components, the models were specified as *ARFIMA*(2,0.4767,18) and *ARIMA*(4,1,15). We estimated the parameters of the model using 900 in-sample data and used this estimates to forecast 70 out-of-sample data. Having Compared the forecasting results of the two models we concluded that the ARFIMA is a much better model in this regard.

Key words: Long memory; ARFIMA model; ARIMA model; Stock price; Tehran stock Exchange.

INTRODUCTION

The last two decades have witnessed tremendous advances in econometrics time series studies. The linear stationary framework of ARMA and VAR models which for many years was the cornerstone of econometric modeling, has increasingly given way to methods that can deal with the manifestly non-stationary and nonlinear features of many economic as well as financial time series data. Two types of models in particular have found their way into the mainstream of applied research. These are the unit root/cointegration framework for non-stationary time series and the ARCH and related models of conditional heteroscedasticity. Recent works has been aimed at both extending our understanding of these well-established models, and widening the range of data features can be handled. Long memory models generalized the unit root model of non-stationary.

Peters [36] notes that most financial markets are not Gaussian in nature and tend to have sharper peaks and fat tails. In the face of such evidence, a number of traditional methods based on Gaussian normality assumption have their own limitations in providing accurate forecasts.

One of the key points explained by Peters [36] is the fact that most financial markets have a very long memory property. In other words, what happens today affects the future forever. This indicates that current data c00orrelated with all past data to varying degrees. This long memory component of the market can not be adequately explained by systems that work with short-memory parameters. Short-memory systems are characterized by using the use of last i values for making the forecast in univariate analysis. For example most statistical methods last i observation is given in order to predict the actual values at time $i+1$.

Traditional models describing short-term memory, such as AR (p), MA (q), ARMA (p, q), and ARIMA (q, d, q), cannot precisely describe long-term memory. A set of models has been established to overcome this difficulty, and the most famous one is the autoregressive fractionally integrated moving average (ARFIMA or ARFIMA(p, d, q)) model. ARFIMA model was established by Granger and Joyeux [25]. Granger[25] made further discussion on this topic. An overall review about long-term memory and ARFIMA model was made by Baillei [2]. Hosking derived the bias, variance, and asymptotic distribution of the sample mean, and autocorrelations of long-term memory time series. Furthermore, he employed these characteristics in ARFIMA model.

An important step in building ARFIMA model is fractional differencing. However, due to difficulties in fractional differencing, most economists use first-order differencing as an alternative. Convenient as it is, such replacement will undoubtedly cause over-differencing, which will lead to the loss of information of the time series.

There are three steps in the procedure of establishing an ARFIMA model. First, testing for long-term memory in the time series, and determining the

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fractional differencing parameter d. Second, imposing fractional differencing on the series and obtaining an ARMA process. Third, determining the other two parameters of ARFIMA model, namely p and q.

The rest of the paper is organized as follows. In section 2 we briefly introduce the empirical studies on long memory and ARFIMA model. The methods of testing long memory and determining differencing parameter are presented in section 3. In section 4 we analyze the underlying data and implement the test for recognition long memory of series and establish ARFIMA model on it. Finally, the conclusions are presented in section 5.

Theoretical and Empirical Studies on Long Memory and Arfima Model: The last two decades of macro- and financial economic studies have resulted in a vast array of important contributions to the area of long-memory modeling, both from a theoretical and an empirical perspective. From a theoretical perspective, much effort has focused on issues of testing and estimation, and a very few important contributions include Granger [25], Granger and Joyeux [25], Hosking, Geweke and Porter-Hudak [26], Lo [28], Sowell [47,48], Ding *et al.* [10], Cheung and Diebold [8], Robinson [38,39], Engle and Smith [16], Diebold and Inoue [11], Breitung and Hassler [6] and Dittman and Granger [12].

The empirical analysis of long-memory models has seen equally impressive treatment, including studies by Diebold and Rudebusch [13,14,15], Hassler and Wolters [18], Hyung and Franses [19], Bos *et al.* [7], Chio and Zivot [9] and van Dijk *et al.,* [52]. The impressive array of papers on the subject is perhaps not surprising, given that long-memory models in economics is one of the many important areas of research that has stemmed from seminal contributions made by Clive W.J. Granger [25].

When the integration parameter d in an ARIMA process is fractional and greater than zero, the process exhibits long memory. Stationary long-memory models ($0<d<0.5$), also fractionally integrated ARMA (ARFIMA) models, have been considered by researchers in many fields.

One motivation for these studies is that many empirical time series have a sample autocorrelation function which declines at a slower rate than for an ARIMA model with finite orders and integer d. The forecasting potential of fitted ARFIMA models, as opposed to forecast results obtained from other time series models, has been a topic of various papers and a special issue. Ray [43,44] undertook such a comparison between seasonal ARFIMA models and standard (non-fractional) seasonal ARIMA models. The results show that higher order AR models are capable of forecasting the longer term well when compared with ARFIMA models. Following Ray [43,44], Smith and Yadav [46] investigated the cost of assuming a unit difference when a series is only fractionally integrated with $d\neq 1$. Over-differencing a series will produce a loss in forecasting performance one-step-ahead, with only a limited loss thereafter. By contrast, under-differencing a series is more costly with larger potential losses from fitting a mis-specified AR model at all forecast horizons. This issue is further explored by Andersson [1] who showed that misspecification strongly affects the estimated memory of the ARFIMA model, using a rule which is similar to the test of Oller [33]. Man [31] argued that a suitably adapted ARMA(2,2) model can produce short-term forecasts that are competitive with estimated ARFIMA models. Multistep-ahead forecasts of long-memory models have been developed by Hurvich [21] and compared by Bhansali and Kokoszka [5]. Many extensions of ARFIMA models and comparisons of their relative forecasting performance have been explored. For instance, Franses and Ooms [17] proposed the so-called periodic ARFIMA(0,d,0) model where d can vary with the seasonality parameter. Ravishanker and Ray [45] considered the estimation and forecasting of multivariate ARFIMA models. Baillie and Chung [3] discussed the use of linear trend-stationary ARFIMA models, while the paper by Beran, Feng, Ghosh and Sibbertsen [4] extended this model to allow for nonlinear trends. Souza and Smith [49] investigated the effect of different sampling rates, such as monthly versus quarterly data, on estimates of the long-memory parameter d. In a similar vein, Souza and Smith [50] looked at the effects of temporal aggregation on estimates and forecasts of ARFIMA processes. Within the context of statistical quality control, Ramjee, Crato, and Ray [42] introduced a hyperbolically weighted moving average forecast-based control chart, designed specifically for non-stationary ARFIMA models.

Fractional Differencing and Long Memory: Most financial time series are non-stationary, with their means and covariance fluctuating in time. Therefore, how to transform a non-stationary time series into a stationary one became an important problem in the field of time series analysis. For a long period of time, it has become a standard practice for time series analysts to consider differencing their time series to achieve stationary time series. However, econometricians were somewhat reluctant to accept this, believing that they may lose something of importance. Take ARFIMA (0, d, 0) as an example. Such a process can be expressed as $(1-L)^d x_t = \varepsilon_t$, often called fractional white noise. When d = 0, x_t is merely a white noise, and its ACF decreases to zero quickly.

When d=1, x_t is a random walk, whose value of ACF is 1, and it can be regarded as a white noise after the first-order differencing. When d is non-integer, the ith element of the fractional differenced time series is actually the weighted sum of $x_i, x_{i-1}, \ldots, x_0$ elements of the original time series. The ith element of the fractional differenced time series is not only determined by x_i and x_{i-1}, but also influenced by all historical data ahead of x_i, this is just the characteristic of long-term memory.

Recognition Methods for Long Memory and Determination of Differencing Parameter: There are various methods such as, rescaled range analysis(R/S), modified rescaled range analysis (MRS), and de-trended fluctuation analysis(DFA), that popularly used for testing long memory. The Hurst rescaled range analysis proposed by Henry Hurst in 1951 to test presence of correlations in time series. The main idea behind the R/S analysis is that one looks at the scaling behavior of the rescaled cumulative deviations from the mean. Consider a time series of length N. time period is divided into m contiguous sub-periods of length n such that $m \times n = N$ and then the mean and standard deviation is calculated for each sub-period. Then the time series of accumulated departures from the mean is calculated for each sub-period and the range of these time series is rescaled by the corresponding standard deviation.

$$R/S = \frac{\left[\underset{0\le k\le n}{Max} \sum_{t=1}^{k} (X_t - \bar{X}_n) - \underset{0\le k\le n}{Min} \sum_{t=1}^{k} (X_t - \bar{X}_n) \right]}{S(n)} \quad (1)$$

Recall that we had m contiguous sub-period of length n, the average of R/S is the R/S(n) value for length n.

The above calculations must be repeated for different time horizons. This is achieved by successively increasing n and repeating the calculation until all integer ns have been covered.

The estimate of the Hurst exponent H is the slop of $\log(R/S(n)) = a + H\log(n)$ regression that can be achieved by performing OLS. If $0<H<1$ then we can conclude that the underlying time series has long memory.

In 1991, Lo introduced a stronger test based on a modified R/S statistic, which is known to be too strong to indicate a true long memory process. The only difference between R/S and MRS values is at denominator of (1) which is as follows:

$$R'/S(n) = \frac{\left[\underset{0\le k\le n}{Max} \sum_{i=1}^{k} (X_i - \bar{X}_n) - \underset{0\le k\le n}{Min} \sum_{i=1}^{k} (X_i - \bar{X}_n) \right]}{\sigma(n)}$$

$$\sigma_n^2(q) = \sigma_x^2(q) + \frac{2}{n}\sum_{j=1}^{q} w_j(q) \left[\sum_{i=j+1}^{n} (x_i - \bar{x}_n)(x_{i-j} - \bar{x}_n) \right]$$

$$w_j(q) = 1 - \frac{j}{q+1} \qquad q < n$$

The process of calculations of $R'/S(n)$ s the same as $R/S(n)$ except that the denominator of $R'/S(n)$ is the root of the sample variance augmented with weighted auto covariance up to a lag determined q. For q=0, this is the same as the R/S statistic. This auto covariance part of denominator is non-zero for series exhibiting short-term memory and this make the statistic robust to heteroscedasticity.

After calculation of $R'/S(n)$ for different size of sub-period, n, the OLS procedure must be implemented on regression $\log(R'/S(n)) = \alpha + H\log(n)$ The slope of the regression, H, is the same as Hurst exponent. If 0<H<1, we can conclude that the series under consideration has long memory.

De-trended fluctuation analysis (DFA), proposed by Peng *et al.,* [34], provides a simple quantitative parameter to represent the correlation properties of a time series. The advantages of DFA over above techniques are that it permits the detection of long-range correlation embedded in seemingly non-stationary time series, and also avoid the spurious detection of apparent long-range correlation that are an artifact of non-stationary.

To implement the DFA, first, the time series must be integrated:

$$x(k) = \sum_{i=1}^{k} (x_i - \bar{x})$$

Where x_t is the t th observations and $\bar{X}$ s the average value of the series. Next, the vertical characteristic scale of the integrated time series is measured. To do so, the integrated time series is divided into m non-overlapping time interval of length n. In each time interval a line is fitted via OLS, which is called the local trend. The x coordinate of the straight line segments is denoted by $\hat{x}(k)$ Then the integrated time series, $x(k)$, is de-trended by subtracting the local trend, $\hat{x}(k)$ in each time interval.

$$x_i = x(k) - \hat{x}(k)$$

For a given interval size n, the characteristic size of fluctuation for this integrated and de-trended time series is calculated by

$$F(n) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i)^2}$$

The above computation is repeated over all time scales to provide a relationship between *F(n)* and *n*. A power law relation between *F(n)* and *n* indicates the presence of scaling that is, $F(n) \approx n^a$. The parameter *a*, called the scaling exponent or correlation exponent, represents the correlation properties of time series and is the same Hurst exponent. If a>0.5, there are positive correlation in time series.

Data Description: We analyze the value of 970 daily closing prices of Tehran Stock Price Index (TSPI) from 26th March 2003 to 8th July 2007. The data are available on www.irbourse.com. The data have shown in figure.1. As the figure.1 exhibits, the TSPI have increased till mid of 2004 and then decreased up to end of the period under consideration.

The ADF statistic has been presented in table 1. It clearly show that the underlying series under consideration is nonstationary in level. Therefore, it must become stationary.

Testing for Long Memory: In this paper we used MRS method for testing long memory in TSPI series. To do this, we divided the TSIP series into sub-periods of length 10 and calculated R'/S for each sub-period. The average of these R'/S is the $R'/S(n)$ corresponding to n=10. The above computation repeated for different size of n. after having calculated $R'/S(n)$ values for a large range of different time horizons n, we plotted $\log(R'/S(n))$ against log*(n)*. It has been illustrated in figure 2. As the figure 2 exhibits, the $R'/S(n)$ increases as *n* goes up.

Applying OLS on $\log(R'/S(n))$ as dependent variable and lig*(n)* as independent variable yielded

$Log(R'/S(n)) = -1.3853 + 0.9767 Log(n)$. Since, the estimate of the slop of regression equation, the Hurst exponent, H, is less than 1, the series exhibits long memory. As peters has proposed, The fractional differencing parameter, d, can be obtained by $d=H$-0.5. Therefore, d=0.4767.

Fractional Differencing of Time Series: To achieve stationary time series, it needed to be fractionally differenced. After d=0.4767 was determined we obtained the fractional differencing time series as follows:

$$w_t = (1-L)^d x_t$$

Where, L is the lag operator, w_t is the fractional differenced time series, and x_i is the initial series (in this paper TSPI). The fractional difference operator is defined as

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \cdots \quad (1)$$

For any real number of d>-1, the relation (1) can be expressed by an hyper geometric function like gamma

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} L^k$$

All steps of the calculation of d and w_t was done using programming techniques in Eviews. Having compared the fractional differenced time series and the first-order differenced time series, we plotted those in figure3.

Figure 3.a and figure 3.b illustrate the fractional differenced and first order differenced time series respectively. According to Figure.3, the differenced time series fluctuated around their means but the first-order differenced time series has large fluctuations whereas, the fractional differenced time series has very small fluctuations. Of course, both of the series are stationary.

Establishing ARFIMA and ARIMA Models: To establish an ARFIMA and ARIMA model, the values of p and q must be determined. First, we computed the values of autocorrelation and partial autocorrelation of series w_t and d(TSPI) which is the first-order differenced time series. There are certainly some other methods in determining parameters p and q, we choose the rule developed by Box and Jenkins in establishment of ARMA models, because this method was the most mature one among others. According to this method, we matched the actual and the theoretical behavior of the autocorrelation of the time series, and found out a best pare of (p, q). The value of autocorrelation and partial autocorrelation could be considered as insignificant if larger than $2/\sqrt{N}$ where N is the total number of observations. In order to comparison of the forecasting performance of the models, we used 900 observations as in-sample data for determination of the parameters p and q of ARFIMA and ARIMA models, and the rest of them as out-of-sample data were used for comparison. The values of ACF and PACF of fractional and first-order differenced time series are illustrated in Table 2 and figure 4.

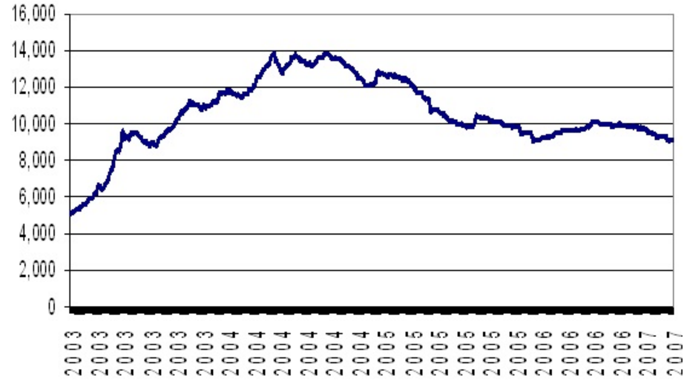

Fig. 1: Tehran Stock Price Index

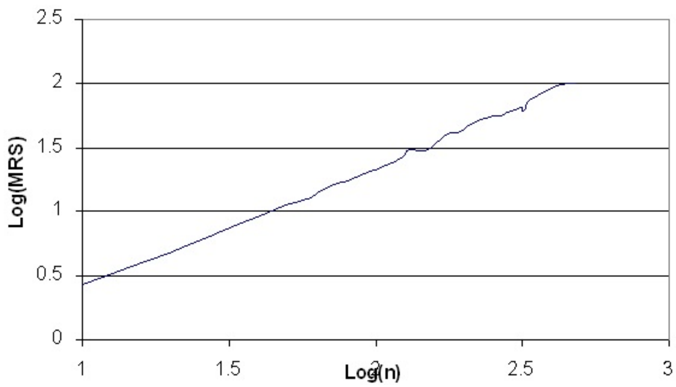

Fig. 2: Log(MRS) against log(n)

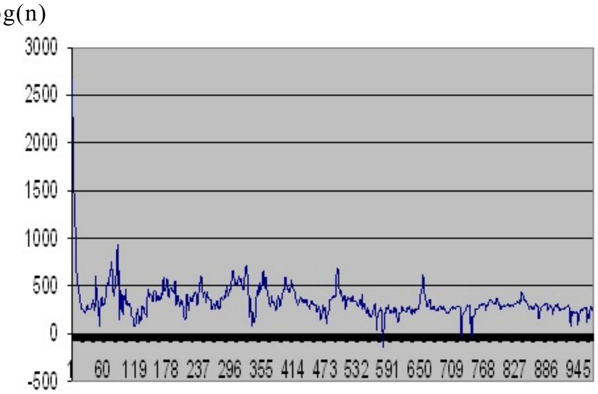

Fig. 3a: fractional differencing time series

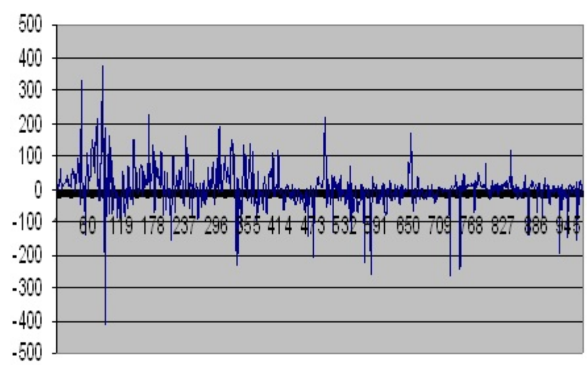

Fig. 3b: first-order differencing time series

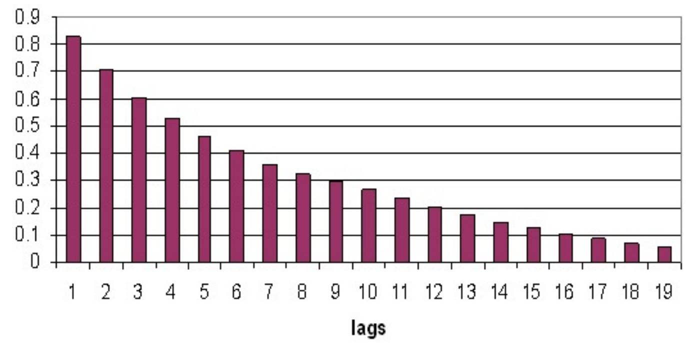

Fig. 4a: ACF of fractional differenced

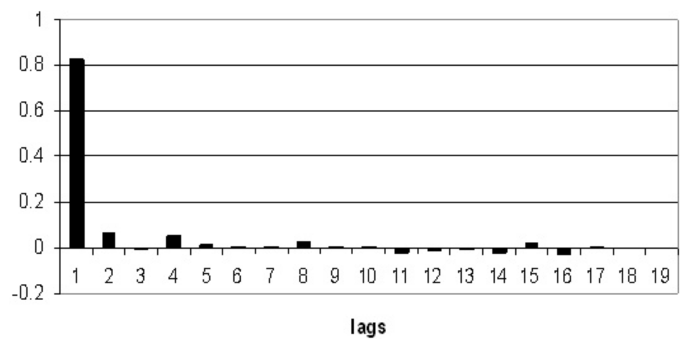

Fig. 4b: PACF of fractional differenced

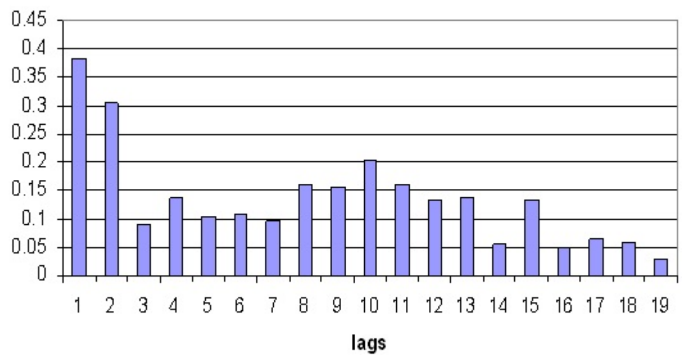

Fig. 4c: ACF offirst-order differenced

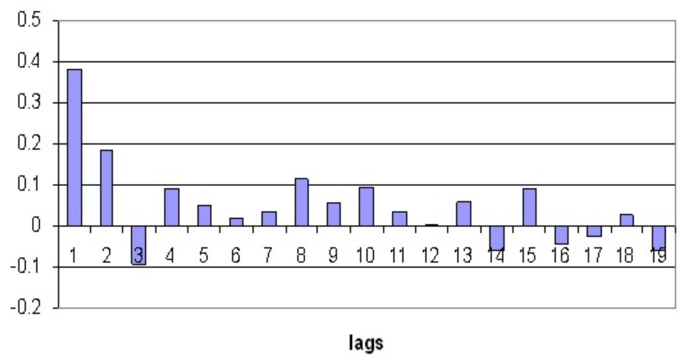

Fig. 4d: PACF offirst-order differenced

Table 1: Augmented Dickey- Fuller Test for TSPI

ADF Test Statistic	0.650554	1% Critical Value*	-2.5678
		5% Critical Value	-1.9397
		10% Critical Value	-1.6158

Table 2: autocorrelation and partial autocorrelation functions

	fractional differenced		first-order differenced	
Lags	ACF	PACF	ACF	PACF
1	0.83	0.83	0.384	0.384
2	0.71	0.066	0.305	0.185
3	0.602	-0.008	0.091	-0.092
4	0.529	0.055	0.136	0.093
5	0.464	0.01	0.105	0.053
6	0.41	0.007	0.108	0.018
7	0.362	0.007	0.095	0.035
8	0.328	0.029	0.161	0.115
9	0.296	0.003	0.156	0.054
10	0.269	0.007	0.204	0.094
11	0.236	-0.021	0.161	0.036
12	0.204	-0.013	0.133	0.002
13	0.175	-0.006	0.136	0.059
14	0.145	-0.023	0.055	-0.061
15	0.127	0.02	0.133	0.091
16	0.102	-0.027	0.05	-0.044
17	0.086	0.006	0.064	-0.026
18	0.072	-0.001	0.058	0.027
19	0.059	-0.004	0.028	-0.06

Since $2/\sqrt{N}$ equals to 0.067, it is obviously clear that the models can be shown as *ARFIMA*(2,0.4767,18) and *ARIMA*(4,1,15) To be more specific, the models could be specified as:

ARFIMA(2,0.4767,18): $\Phi(L)(1-L)^{0.4767}x_t = \Theta(L)\varepsilon_t$

Where the polynomials $\Phi(L) = 1-\phi_1 L-\phi_2 L^2$ and $\Theta(L) = 1-\theta_1 L-\theta_2 L^2-\cdots-\theta_{18} L^{18}$ are AR and MR component operators respectively and $(1\text{-}L)^{0.4767}$ is the fractional differencing operator. The differences between functional forms of the models are in orders of the polynomials and the power of differencing operator. Therefore,

ARIMA(4,1,15):

$$(1-\phi_1 L-\cdots-\phi_4 L^4)(1-L)x_t = (1-\theta_1 L-\cdots-\theta_{15} L^{15})\varepsilon_t$$

The estimated values of parameters were obtained by Eviews which have been provided in Table 5.

Forecasting Results: After parameters were determined, we made a 70-steps ahead forecast with *ARFIMA*(2,0.4767,18) and *ARIMA*(4,1,15), and compared the forecasting values with the real observations. The results have illustrated in table 6 and figure 5.

As the figure.5 obviously shows, the values that have forecasted with ARFIMA model are closest to real data than those of ARIMA model. Therefore, we can conclude that the forecasting performance of ARFIMA model is highly better than that of the ARIMA model.

Table 5: the values of parameters of the models

ARFIMA

$$\Phi(L) = 1 + 1.7416\ L - 0.742\ L^2$$

$$\Theta(L) = 1 - 0.889\ L + 0.113\ L^2 - 0.255\ L^3 + 0.151\ L^4 - 0.059\ L^5 + 0.023\ L^6 - 0.048\ L^7 + 0.084\ L^8 - 0.032\ L^9 + $$
$$+ 0.08\ L^{10} - 0.086\ L^{11} + 0.015\ L^{12} - 0.014\ L^{13} - 0.104\ L^{14} + 0.148\ L^{15} - 0.143\ L^{16} + 0.056\ L^{17} - 0.034\ L^{18}$$

ARIMA

$$\Phi(L) = 1 + 0.687\ L + 0.754\ L^2 + 0.06\ L^3 - 0.503\ L^4$$

$$\Theta(L) = 1 - 0.368\ L - 0.703\ L^2 - 0.496\ L^3 + 0.354\ L^4 + 0.134\ L^5 + 0.106\ L^6 - 0.055\ L^7 + 0.085\ L^8 + 0.025\ L^9 + $$
$$+ 0.063\ L^{10} - 0.079\ L^{11} - 0.05\ L^{12} - 0.03\ L^{13} - 0.078\ L^{14} + 0.098\ L^{15}$$

Table 6: the forecasted values with ARFIMA and ARIMA and the real observations for 70 out-of-sample data

No.	forecasting with ARFIMA	forecasting with ARIMA	real observations	No.	forecasting with ARFIMA	forecasting with ARIMA	Real observations
901	9789.746	9729.186	9832	937	9508.361	8712.698	9496
902	9813.554	9747.993	9837	938	9504.431	8679.693	9476
903	9833.701	9726.192	9838	939	9479.99	8636.6	9454
904	9822.638	9682.254	9847	940	9469.931	8606.522	9454
905	9826.299	9667.532	9858	941	9463.128	8575.366	9457
906	9848.146	9641.079	9864	942	9465.109	8548.247	9433
907	9840.999	9608.748	9877	943	9463.806	8534.501	9413
908	9860.104	9591.256	9836	944	9411.163	8477.188	9270
909	9828.718	9550.822	9843	945	9316.087	8418.515	9263
910	9824.648	9518.157	9792	946	9289.081	8372.6	9270
911	9804.492	9496.627	9792	947	9325.751	8365.709	9285
912	9774.421	9443.071	9782	948	9316.988	8341.355	9292
913	9792.144	9436.903	9776	949	9329.071	8320.301	9302
914	9775.901	9405.467	9774	950	9340.831	8306.352	9303
915	9771.914	9369.74	9766	951	9333.666	8281.567	9290
916	9768.002	9357.661	9757	952	9319.226	8248.582	9292
917	9757.624	9319.976	9747	953	9314.923	8219.783	9295
918	9738.615	9284.891	9755	954	9315.524	8190.84	9289
919	9748.67	9267	9764	955	9313.738	8168.913	9298
920	9755.52	9236.201	9752	956	9323.85	8150.341	9308
921	9737.951	9206.474	9755	957	9332.538	8136.281	9309
922	9747.882	9184.463	9759	958	9342.255	8126.671	9303
923	9743.16	9153.489	9759	959	9321.431	8094.481	9291
924	9751.609	9136.132	9756	960	9319.865	8074.71	9312
925	9740.902	9104.456	9765	961	9340.282	8063.507	9155

Table. 6: Continue

926	9750.876	9079.606	9753	962	9206.059	7995.496	9156
927	9748.272	9061.236	9758	963	9185.191	7951.772	9114
928	9743.904	9029.566	9760	964	9188.52	7945.555	9067
929	9754.893	9013.226	9561	965	9120.163	7895.956	9084
930	9580.99	8927.819	9564	966	9135.439	7876.991	9104
931	9559.463	8869.594	9575	967	9162.609	7872.269	9128
932	9613.759	8882.338	9580	968	9176.798	7860.443	9148
933	9593.302	8849.758	9515	969	9180.146	7836.086	9155
934	9534.847	8799.658	9528	970	9191.769	7820.765	9137
935	9539.946	8773.642	9532	971	9160.837	7780.561	9158
936	9564.503	8766.884	9503				

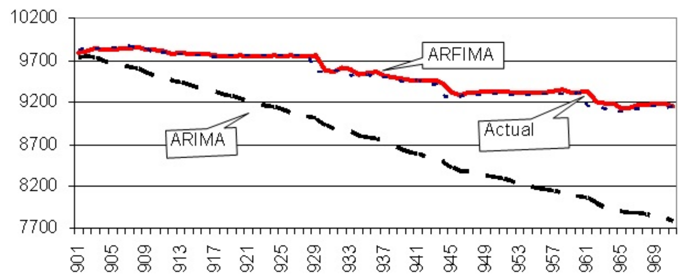

Fig. 5: Comparing the forecasting performanca of the ARFIMA and ARIMA

Conclusion Remarks: In this paper we studied the long memory property of Tehran Stock Price Index via MRS analysis. We obtained a Hurst exponent H=0.9767, indicating that the TSPI time series has comparatively strong long memory. Then, we calculated the fractional differenced time series using fractional differencing parameter $d=H-0.5=0.4767$ as Peters has proposed. To establish an ARFIMA and ARIMA models on underlying series and determine the parameters of the models, we followed the rule of the Box and Jenkins, using the values of the autocorrelation and partial autocorrelation functions of differenced time series for 900 in-sample data. This provided *ARFIMA*(2,0.4767,18) and ARIMA(4,1,15). Then we estimated the parameters of the AR and MR operators of these models. We used these estimates for making 70-steps ahead forecast with models, and compared the forecasting values with real observations. The results showed that the forecasting performance of the ARFIMA model is strongly better than that of ARIMA model.

REFERENCES

1. Anderson, M.K., 2000. Do long-memory models have long memory? International Journal of Forecasting, 16: 121-124.
2. Baillie, R.T., 1996. Long memory processes and fractional integration in econometrics- J. Econometrics, 73: 5-59.
3. Baillie, R.T. and S.K. Chung, 2002. Modeling and forecasting from trend-stationary long memory models with applications to climatology. International Journal of Forecasting, 18: 215-226.
4. Beran, J., Y. Feng, S. Ghosh, P. Sibbertsen, 2002. On robust local polynomial estimation with long-memory errors. International Journal of Forecasting, 18: 227-241.
5. Bhansali, R.J. and P.S. Kokoszka, 2002. Computation of the forecast coefficients for multistep prediction of long-range dependent time series. International Journal of Forecasting, 18: 181-206.
6. Breitung, J., U. Hassler, 2002. Inference on the cointegration rank in fractionally integrated processes. Journal of Econometrics, 110: 167-185.
7. Bos, C.S., P.H. Franses, M. Ooms, 2002. Inflation, forecast intervals and long-memory regression models. International Journal of Forecasting, 18: 243-264.
8. Cheung, Y.W., F.X. Diebold, 1994. On maximum likelihood estimation of the difference parameter of fractionally integrated noise with unknown mean. Journal of Econometrics, 62: 301-316.

9. Chio, K., E. Zivot, 2002. Long memory and structural changes in the forward discount: an empirical investigation. Working Paper, University of Washington.
10. Ding, Z., C.W.J. Granger, R.F. Engle, 1993. A long-memory property of stock returns and a new model. Journal of Empirical Finance., 1: 83-106.
11. Diebold, F., A. Inoue, 2001. Long memory and regime switching. Journal of Econometrics, 105: 131-159.
12. Dittman, I., C.W.J. Granger, 2002. Properties of nonlinear transformations of fractionally integrated processes. Journal of Econometrics, 110: 113-133.
13. Diebold, F.X., G.D. Rudebusch, 1989. Long memory and persistence in aggregate output. Journal of Monetary Economics, 24: 189-209.
14. Diebold, F.X., G.D. Rudebusch, 1991a. Is consumption too smooth? Long memory and the Deaton paradox. Review of Economics and Statistics., 73: 1-9.
15. Diebold, F.X., G.D. Rudebusch, 1991b. On the power of the Dickey–Fuller test against fractional alternatives. Economics Letters, 35: 155-160.
16. Engle, R.F., A.D. Smith, 1999. Stochastic permanent breaks. Review of Economics and Statistics, 81: 553-574.
17. Franses, P.H. and M. Ooms, 1997. A periodic long-memory model for quarterly UK inflation. International Journal of Forecasting, 13: 117-12.
18. Hassler, U., J. Wolters, 1995. Long memory in inflation rates: international evidence. Journal of Business and Economic Statistics, 13: 37-45.
19. Hyung, N., P.H. Franses, 2001. Structural breaks and long-memory in US inflation rates: do they matter for forecasting? Working Paper, Erasmus University.
20. Hurst, H.R., 1951. Long-term storage in reservoirs, Trans, Amer, Soc, Civil Eng., 116: 770-799.
21. Hurvich, C.M., 2002. Multi-step forecasting of long memory series using fractional exponential models. International Journal of Forecasting, 18: 167-179.
22. Hiemstra, C. and J.D. Jones, 1999. Another look at long memory in common stock returns- J. Empir. Finance., 6: 373-401.
23. Iglesias Pilar, *at el.,* 2006. data analysis using regression models with missing observations and long memory: an application study- Computational statistics & data analysis-50.
24. Jacobson, B., 1996. Long term dependence in stock returns- J. Empir. Finance 3.
25. Granger C.W.J. and R. Joyeux, 1980. An introduction to long-memory time series models and fractional differencing- J. Time Series Anal., 15-29.
26. Geweke, J., S. Porter-Hudak, 1983. The estimation and application of long-memory time series models. Journal of Time Series Analysis, 4: 221-238.
27. Guest editional, 2002. long memory and nonlinear time series- Journal of Econometrics, 110.
28. Lo, A.W., 1991. long-term memory in stock market price- Econometrica, 59(5): 1279-1313.
29. Norouzzadeh, P. and G.R. Jafari, 2005. application of multifractal measures to Tehran price index- Physica A, 356: 609-627.
30. Norouzzaheh, P and B. Rahmani, 2006. a multifractal detrended fluctuation description of Iranian Rial-US dollar exchange rate- Physica A., 367.
31. Man, K.S., 2003. Long memory time series and short term forecasts. International Journal of Forecasting, 19: 477-491.
32. Morana, C., 2006. multivariate modeling of long memory prosesses with common components- Computational statistics & data analysis.
33. Oller, L.E., 1985. How far can changes in general business activity be forecasted? International Journal of Forecasting, 1: 135-141.
34. Peng, C.K., S. Havlin, H.E. Stanley, A.L. Goldberger, 1995. quantification of scaling exponent and crossover phenomena in nonstationary heartbeat time series- Chaos, 5: 82-87.
35. Pan Ming-Shium, Y.A., K.C. Liu, Chan, 1996. An examination of long-term dependence in black market exchange rates in eight Pacificbasin countries- Int. Rev. Econ. Finance., 5: 175-185.
36. Peters, E.E., 1991. fractal market analysis- Wiley-New York.
37. Peters, E.E., 1999. Chaos and Order in the Capital Markets, first ed- Economics Science Press-Beijing.
38. Robinson, P., 1995a. Log-periodogram regression of time series with long range dependence. The Annals of Statistics, 23: 1048-1072.
39. Robinson, P., 1995b. Gaussian semi-parametric estimation of long range dependence. The Annals of Statistics, 23: 1630-1661.
40. Rodriguez Eduardo, *at el.,* 2006. de-trended fluctuation analysis of heart intrabeat dynamics- Phisica A.
41. Ramirez Jose Alvares, *at el.,* 2005. de-trending fluctuation analysis based on moving average filtering- Phisica A- 354.
42. Ramjee, R., N. Crato, and B.K. Ray, 2002. A note on moving average forecasts of long memory processes with an application to quality control. International Journal of Forecasting, 18: 291-297.
43. Ray, B.K., 1993a. Long-range forecasting of IBM product revenues using a seasonal fractionally differenced ARMA model. International Journal of Forecasting, 9: 255-269.
44. Ray, B.K., 1993b. Modeling long-memory processes for optimal long-range prediction. Journal of Time Series Analysis, 14: 511-525.

45. Ravishanker, N., and B.K. Ray, 2002. Bayesian prediction for vector ARFIMA processes. International Journal of Forecasting, 18: 207-214.
46. Smith, J., and S. Yadav, 1994. Forecasting costs incurred from unit differencing fractionally integrated processes. International Journal of Forecasting, 10: 507-514.
47. Sowell, F.B., 1992a. Maximum likelihood estimation of stationary univariate fractionally integrated time series models. Journal of Econometrics, 53: 165-188.
48. Sowell, F.B., 1992b. Modeling long-run behavior with the fractional ARIMA model. Journal of Monetary Economics, 29: 277-302.
49. Souza, L.R. and J. Smith, 2002. Bias in the memory for different sampling rates. International Journal of Forecasting, 18: 299-313.
50. Souza, L.R. and J. Smith, 2004. Effects of temporal aggregation on estimates and forecasts of fractionally integrated processes: A Monte-Carlo study. International Journal of Forecasting, 20: 487-502.
51. Tang Ta-Lun, Shwu-Jane Shieh, 2005. long memory in stock index futures markets: A value-at-risk approach- Physica A., 366: 437-488.
52. van Dijk, D., P. Franses, R. Paap, 2002. A nonlinear long-memory model, with an application to US unemployment. Journal of Econometrics, 110: 135-165.
53. Xiu Jin and Yao Jin, 2007. empirical study of ARFIMA model based on fractional differencing- Physica A- 377.
54. Yin-Wong Chueng Yin-Wong and K.S. Lai, 1995. A search for long memory in international stock market returns- J. Int. Money Finance 14.