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Journal of Applied Sciences Research, 7(3): 229-235, 2011
ISSN 1819-544X
This is a refereed journal and all articles are professionally screened and reviewed

ORIGINAL ARTICLES

On The Weighted Intervals of Fuzzy Numbers

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ABSTRACT

This article proposes a novel approach to ranking fuzzy numbers based on WLUPM (weighted lower and upper possibilistic means). At first, by using this significance, we will compose WIVPM (weighted interval-value possibilistic mean), that this interval can be used as a crisp set approximation with respect to a fuzzy quantity. Therefore, with that, we define a method for ordering. This method can effectively rank various fuzzy numbers, their images and overcome the shortcomings of the previous techniques.

Key words: Ranking; Fuzzy numbers; Defuzzification; WLUPM; WIVPM.

Introduction

In decision analysis under fuzzy environment, ranking fuzzy numbers is a very important. Since many authors introduced the relevant concepts of fuzzy numbers, many researchers have proposed the related methods or applications for ranking fuzzy numbers. For instance, Bortolan et al. reviewed some methods to rank fuzzy numbers in 1985, Chen and Hwang proposed fuzzy multiple attribute decision making in 1972, Choobineh and Li proposed an index for ordering fuzzy numbers in 1993, Dias ranked alternatives by ordering fuzzy numbers in 1993, Lee ranked fuzzy numbers with a satisfaction function in 1998, Requena utilized artificial neural networks for the automatic ranking of fuzzy numbers in 1994, Fortemps presented ranking and defuzzification methods based on area compensation in 1996, and Raj *et al.* investigated maximizing and minimizing sets to rank fuzzy alternatives with fuzzy weights in 1999. In recent years, many methods are proposed for ranking different types of fuzzy numbers (Saneifard *et al* 2007, Saneifard 2009, Abbasbandy and Hajjary 2009, Wang and Kerre 2001), and can be classified into four major classes: preference relation, fuzzy mean, and spread fuzzy scoring, and linguistic expression. But each method appears to have advantages as well as disadvantages. Having reviewed the previous methods, this article proposes a novel method to find the order of fuzzy numbers. Representing fuzzy numbers by proper intervals is an interesting and important problem. Besides, an interval representation of a fuzzy number may have many useful applications. By using such a representation, it is possible to apply approaches in fuzzy numbers from which some results derived in the field of interval number analysis. Many authors have studied the crisp set approximation of fuzzy sets. They proposed a rough theoretic definition of that crisp approximation which is the nearest interval approximation and the nearest ordinary set of a fuzzy set. Based on the reasons mentioned above, this article proposes a conceptual procedure and a method to use the concept of WIVPM in order to find the order of fuzzy numbers. The advantage of this method is that can distinguish the alternatives clearly. The main purpose of this article is that the interval value can be used as a crisp set approximation of a fuzzy number, in which the researchers obtain a crisp set approximation with respect to a fuzzy quantity, and then define a method for ordering of fuzzy numbers. Therefore, by means of this defuzzification, this article aims to present a novel method for ranking of fuzzy numbers.

The paper is organized as follows: In Section 2, some fundamental results on fuzzy numbers are recalled. In Section 3, a crisp set approximation of a fuzzy number (WIVPM) is introduced. The Proposed method for ranking fuzzy numbers is mentioned in the Section 4. Discussion and comparison of this work and other methods are carried out in Section 5. The paper ends with conclusions in section 6.

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2. Basic Definition and Notations

The basic definition of a fuzzy number are given in (Heilpern, 1992; Kauffman *et al*., 1991) as follows:

Definition 2.1.

A fuzzy number *A* is a mapping $\mu_A(x): \Re \rightarrow [0;\ 1]$ with the following properties:

1. μ_A is an upper semi-continuous function on $\Re$,
2. $\mu_A(x) = 0$ outside of some interval $[a, d] \subset \Re$.
3. There are real numbers a,b,c and d such that $a \leq b \leq c \leq d$ and

3.1 $\mu_A(x)$ is a monotonic increasing function on $[a, b]$,

3.2 $\mu_A(x)$ is a monotonic decreasing function on $[c, d]$,

3.3 $\mu_A(x) = 1$ for all x in $[b, c]$.

The set of all fuzzy numbers is denoted by F.

Definition 2.2.

We will use the following LR representation of a fuzzy number A∈F:

$$A = \bigcup_{\gamma\in[0,1]}(\gamma, A^{\gamma})$$

where $\forall\gamma \in [0,1]: A^{\gamma} = [L_A(\gamma), R_A(\gamma)] \subset (-\infty,\infty)$ are γ-cut sets of A∈F. Here, $L: [0,1] \rightarrow (-\infty,\infty)$ is a monotonically non-decreasing and $R: [0,1] \rightarrow (-\infty,\infty)$ is a monotonically non-increasing left-continuous function.

Let $\Re$ be the set of all real numbers. We assume a fuzzy number A that can be expressed for all x∈$\Re$ in the form

$$A(x) = \begin{cases} g(x), & \text{when} \quad a \leq x < b, \\ 1, & \text{when} \quad b \leq x \leq c, \\ h(x) & \text{when} \quad c < x \leq d, \\ 0 & \quad\quad \text{otherwise.} \end{cases} \tag{1}$$

where a,b,c and d are real numbers such that a < b ≤ c < d and g, h are real valued functions such that g is increasing and right continuous and h is decreasing and left continuous. Notice that (1) is an LR fuzzy number. A normal fuzzy number A with shape function g and h defied by

$$g(x) = \left(\frac{x-a}{b-a}\right)^n \tag{2}$$

and

$$h(x) = \left(\frac{d-x}{d-c}\right)^n \tag{3}$$

respectively, where n > 0, will be denoted by A = (a, b, c, d). If n = 1, we simply write A = (a, b, c, d), which is known as a trapezoidal fuzzy number. If n ≠ 1, a fuzzy number A* = (a, b, c, d) is a concentration of A. If 0< n < 1, then A* is a dilation of A. Concentration of A by n = 2 is often interpreted as the linguistic hedge “very”. Dilation of A by n = 0.5 is often interpreted as the linguistic hedge “more or less”. More about linguistic hedges can be found in (Zadeh, 1972).

Each fuzzy number A described by (1) has the following γ-level sets $A_\gamma = [a_\gamma, b_\gamma]$ that $a_\gamma, b_\gamma \in \Re$ and $\gamma \in [0,1]$,

1. $A_\gamma = [g^{-1}(\gamma), h^{-1}(\gamma)]$ for all $\gamma \in (0,1)$,
2. $A_0 = [b, c]$,
3. $A_1 = [a, d]$.

If $A = \langle a, b, c, d \rangle_n$ then for all $\gamma \in [0,1]$,

$$A_\gamma = [a + \gamma^{\frac{1}{n}}(b - a), d - \gamma^{\frac{1}{n}}(d - c)] \qquad (4)$$

In this paper, we will always refer to fuzzy number A described by (1).

2.1. The measure of interval number

The measure of interval is given first which is different from the measure of traditional interval number, such as the length of interval number.

Generally, interval number is denoted as $A(a_1, a_2) = [a_1, a_2]$, where a_1 and a_2 are respectively called left end point and right end point, $a_1 \leq a_2$. Particularly, if $a_1 = a_2$, $A(a_1, a_2)$ denotes real number a_1 . Let, $A(a_1, a_2)$ and $B(b_1, b_2)$ are arbitrary interval numbers, herein $A(a_1, a_2) = B(b_1, b_2)$ if and only if $a_1 = b_1$ and $a_2 = b_2$.

Definition 2.1.1. (Yang et al. 2002)

Let, $A(a_1, a_2)$ is arbitrary interval number. The measure of interval number A define as follows:

$$M_I(A) = \operatorname{sign}(a_1).|a_1.a_2| \qquad (5)$$

Note that, the geometric meaning of the measure that we defined here is monotone function of a triangle area which is constituted be segment $l(a_1, a_2)$ and two axes. The meaning of symbol function is that we can compare the size between two interval numbers when the end point of interval numbers is negative numbers.

3. Weighted Lower and Upper Possibilistic Means

Various authors have studied the mean interval of a fuzzy number, also called the interval-valued mean (Dubois *et al*., 1987; Carlsson, *et al*., 2001). Carlsson and Fuller (2001) defined the lower and upper possibilistic mean values of fuzzy number A with γ-level sets

$$M_*(A) = \frac{\int_0^1 \mathrm{Pos}[A \leq a_\gamma] a_\gamma d\gamma}{\int_0^1 \mathrm{Pos}[A \leq a_\gamma] d\gamma} = 2\int_0^1 \gamma a_\gamma d\gamma, \qquad (6)$$

and

$$M_*(A) = \frac{\int_0^1 \mathrm{Pos}[A \geq b_\gamma] b_\gamma d\gamma}{\int_0^1 \mathrm{Pos}[A \geq b_\gamma] d\gamma} = 2\int_0^1 \gamma b_\gamma d\gamma, \qquad (7)$$

where Pos denotes possibility i.e.

$$\mathrm{Pos}[A \le a_\gamma] = \Pi\left(\left(-\infty, a_\gamma\right]\right) = \gamma$$

$$\mathrm{Pos}[A \ge b_\gamma] = \Pi([b_\gamma, +\infty)) = \gamma$$

Furthermore, Carlsson and Fuller (2001) also introduced the crisp possibilistic mean as

$$\overline{M}(A) = \int_0^1 \gamma\, [a_\gamma + b_\gamma] d\gamma$$

On the basis of (Carlsson, *et al*., 2001), Fuller and Majlender (Full´er, R. and Majlender, P., 2003) further introduced the weighted lower and upper possibilistic means of fuzzy number A with $A_\gamma = [a_\gamma, b_\gamma]$ as

$$M_{*f}(A) = \int_0^1 f(\gamma)\, a_\gamma\, d\gamma = \frac{\int_0^1 f(\mathrm{Pos}[A \le a_\gamma]) a_\gamma d\gamma}{\int_0^1 f(\mathrm{Pos}[A \le a_\gamma]) d\gamma} \tag{8}$$

and

$$M_f^*(A) = \int_0^1 f(\gamma) b_\gamma\, d\gamma = \frac{\int_0^1 f(\mathrm{Pos}[A \ge b_\gamma]) a_\gamma d\gamma}{\int_0^1 f(\mathrm{Pos}[A \ge b_\gamma]) d\gamma} \tag{9}$$

where $f: [0,1] \to \Re$ is said to be a weighted function if f in non-negative and satisfies the following normalization condition $\int_0^1 f(\gamma) d\gamma = 1$.

Note that if $g: [0,1] \to \Re$ be a function non-negative, monotone increasing such that $\int_0^1 g(\gamma)\, d\gamma = S$, then we consider $f(\gamma) = \frac{1}{S} g(\gamma)$. In practical cases, it may be assumed that $f(\gamma) = (k + 1)\gamma^k, k = 0, 1, 2, \cdots$ if $f(\gamma) = 2\gamma$, then

$$M_{*f}(A) = M_*(A)$$

$$M_f^*(A) = M^*(A)$$

Clearly, the weighted lower and upper possibilistic means can be considered as a generalization of the lower and upper possibilistic means in (Carlsson, *et al*., 2001).

4. Comparison of fuzzy Numbers Based On WIVPM

In this section, the researchers will propose the ranking of fuzzy numbers associated with the WIVPM.

Definition 4.1. (Carlsson, et al., 2001).

Let A be a fuzzy number characterized by (1). Let $M_{*f}(A)$ and $M_f^*(A)$ be the WLUPM of fuzzy number A with $A_\gamma = [a_\gamma, b_\gamma]$. Then the interval $M(A) = [M_{*f}(A), M_f^*(A)]$ is called the weighted interval-valued possibilistic mean (WIVPM) of A.

Suppose we want to approximate a fuzzy number by a crisp interval. Thus, the researchers have to use an operator $M: F \to (\text{set of closed intervals in } \Re)$ which transforms fuzzy numbers into family of closed intervals on the real line. Operator M is an interval approximation operator because for any A∈F.

(a) $M(A) \subseteq \text{supp}A$,

(b) $\text{Core}A \subseteq M(A)$.

Definition 4.2.

Let A be an arbitrary fuzzy number and $M(A) = [M_{*f}(A), M_f^*(A)]$ be its WIVP. According to definition 2.1.1, the measure of $M_I(M(A))$ which is an interval number is as $M_I(M(A)) = \text{sign}(M_{*f}(A))|M_{*f}(A).M_f^*(A)|$. We define the measure of fuzzy number A as follows:

$$\text{meas}(A) = \int_0^1 f(\gamma)\, M_I(M(A))d\gamma \tag{10}$$

Obviously, if the fuzzy numbers become interval numbers, then $\text{meas}(A)$ will be the measure of the interval number which can be denoted as $M_I(A)$. For a certain fuzzy numbers, we can obtain $\text{meas}(A)$ by definite integral. But it is not easy to compute definite integral sometimes. For trapezoid fuzzy numbers and triangular fuzzy numbers, the calculation formulas for the indices are given in the paper.

Proposition 4.1.

If $A = \langle a, b, c, d\rangle$ is a trapezoidal fuzzy number, the $\text{meas}(A)$ can be denoted as follows:

$$\text{meas}(A) = \frac{1}{9}(2ac + ad + 4bc + 2bc) \tag{11}$$

Since ever meas can be used as a crisp approximation of a fuzzy number, therefore, the resulting value is used to rank the fuzzy numbers. Thus, meas (A) is used to rank fuzzy numbers.

Let A and B∈F be two arbitrary fuzzy numbers, and meas (A) and meas (A) be the measures of A and B, respectively. Define the ranking of A and B by meas (.) on F, i.e.

1. $\text{meas}(A) = \text{meas}(B)$ if only if $A \sim B$,
2. $\text{meas}(A) < meas(B)$ if only if $A \prec B$
3. $\text{meas}(A) > meas(B)$ if only if $A \succ B$.

Then, this article formulates the order $\gtrsim$ and $\lesssim$ as $A \gtrsim B$ if and only if $A \succ B$ or $A \sim B$, $A \lesssim B$ if and only if $A \prec B$ or $A \sim B$.

Remark 4.1.

If $A \lesssim B$, then $-A \gtrsim -B$.

Hence, this article can infer ranking order of the images of the fuzzy numbers.

To present rationality of this method, some examples are proposed to illustrate these methods and compared with others method. (Saneifard, 2010; Saneifard *et al.* 2007; Ezatti, *et al*. 2010; Allaviranloo, *et al.* 2010, Allaviranloo, *et al*. 2011).

Example 4.1.

Consider the data used in (Saneifard, 2009), i.e. the three fuzzy numbers A= (5,6,6,7), B= (5,9,6,6,7) and C=(6,6,6,7). According to Eq. (11), the ranking index values are obtained i.e. meas(A) = 34.5, meas(B) = 36.4 and meas(C)=36.66. Accordingly, the ranking order of fuzzy numbers is C≻B≻A. However, by (Chu and Tsao, 2002)'s approach, the ranking order is B≻C≻A. Meanwhile, using CV index proposed, the ranking order is A≻B≻C. It is easy to see that the ranking results obtained by the existing approaches (Chu and Tsao, 2002; Cheng, 1999) are unreasonable and are not consistent with human intuition. On the other hand, in (Abbasbandy and Asady, 2006), the ranking result is C≻B≻A, which is the same as the one obtained by the writers approach. However, their approach is simpler in the computation procedure. Based on the analysis results from (Abbasbandy and Asady 2006), the ranking results using their approach and other approaches are listed in Table 1.

Table 1: Comparative results of Example (4.1)

Fuzzy number	New approach	Sign Distance with p=1	Sign Distance with p=2	Chu-Tsao	Cheng Distance	C V index
A	34.5	6.12	8.52	3.00	6.02	0.028
B	36.4	12.45	8.82	3.12	6.34	0.009
C	36.66	12.50	8.85	3.08	6.35	0.008
Results	C≻B≻A	C≻B≻A	C≻B≻A	B≻C≻A	C≻B≻A	B≻C≻A

The above example shows that the results of this method are reasonable results. This method can overcome the shortcoming of other methods.

5. Conclusion

In this peper, at first, we will compose WIVPM, that this interval can be used as a crisp set approximation with respect to a fuzzy quantity. Therefore, with that, we define a method for ordering fuzzy numbers. In this study some preliminary results on properties of such defuzzification reported.

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