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ORIGINAL ARTICLES

Simple Diffusion Model From A Point Source Using Power Law Of Wind Speed

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ABSTRACT

In the present paper, a model for the diffusion of material from a point source in an urban atmosphere is incorporated. The plume is assumed to have a well-defined edge at which the concentration falls to zero. The vertical wind shear is estimated using power law, by employing most of the available techniques of stability categories. The concentrations estimated from the model were compared favorably with the field observations of investigators.

Key words: Urban Atmosphere/ vertical wind shear/ power law/ Stability Categories.

Introduction

Most models for urban air pollution are based on Gaussian plume diffusion or Sutton's equations or the K-theory and require a digital computer. There are some simpler models such as Miller and Holzworth (1967), Hanna (1982), Anon (1989) and Khaled *et al*. (2006). In these models the assumptions used, for example uniform wind, uniform mixing in the mixing layer, ground terrain, etc. are not very realistic and, therefore, these assumptions restrict the use of the model for some special cases which don't normally occur in real life.

The purpose of this study is to suggest a simple physical realistic model which depends on the height and stability of the atmosphere over which diffusion of pollution takes place; also, wind speed is treated as a function of height and stability of the atmosphere. This definitely more closely represents real life situations than does treating wind as a constant quantity. The problem of diffusion and advection of conservative material as it travels downwind is investigated

2-Proposed model and its components:

A mean wind direction is normal to point source with height "h" situated at the ground level and emission rate, Q in studying diffusion and advection of air pollutant. This problem is in two –dimensional in nature because homogeneity in the lateral direction is assumed. Fig. (1) describes the coordinate system direction of the mean wind. The effective height denoted by $H = h_s + \Delta h$, where h_s is the stack height and Δh is the plume height which increases as the plume travel downwind. The analysis that follows assumes steady-state conditions; then the variables; for example the mean wind $\bar{u}(z)$, stability of the atmosphere, source strength doesn't change in the time interval of interest. The ground surface is treated as a complete reflector of matter; that is, no removal occurs.

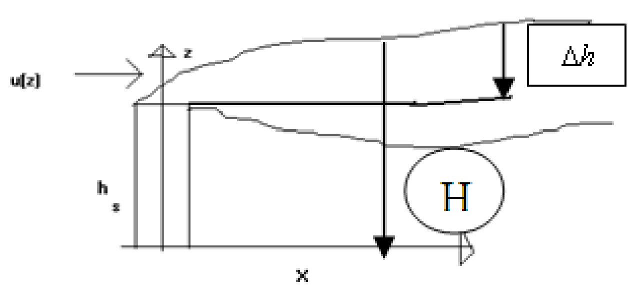

Fig. 1

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Approach used:

A very simple approach, namely the principle of conservation of mass with a steady state can be written as:

$$Q = \int_0^H \bar{u}(z)C(z)dz \quad (1)$$

where

$\bar{u}(z)$ is the mean wind speed,

C(z)is the concentration of material, and

H is the effective height of the plume.

In subsequent sections of this paper the different variables in equations (1), namely, the wind profile, concentration profile and effective height of the plume will be discussed in detail. The integration of equation (1) will lead to the mathematical model.

3- Wind Profile:

Power wind law:

The average value of the wind speed over the plume depth is generally recognized. In practice, the wind speed at the effective release height (h) of the plume is used. Sometimes observation of this wind speed is available, but usually the wind speed must be estimated by using observation near the surface. The power law formula can be used:

$$\bar{u} = u_1\left(\frac{z}{10}\right)^n \quad (2)$$

where z is the height in meters and u_1 is the observed wind speed at a height 10 m. This formula is used by several of the U.S. Environmental Protection Agency (EPA) models, with values of the parameter n estimated by Irwin (1979) given in table 1

Table 1: Estimates of the Power (n) in Eq. 2 for the Six Stability Classes and two Roughnesses

	Stability class					
	A	B	C	D	E	F
Urban n	0.15	.15	0.2	0.25	0.4	0.6
Rural n	0.07	0.07	0.01	0.15	0.35	0.55

4-Height of Plume:

The problem of turbulent diffusion defined in section 2 was first treated by Reborts (1923). He solved the partial differential equation

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial z}\left(K(z)\frac{\partial C}{\partial z}\right) \quad (3)$$

subject to the boundary conditions

$C \rightarrow 0$ and $X \rightarrow \infty$

$K(z)\frac{\partial C}{\partial z} \rightarrow 0$ as $z \rightarrow 0$

the ground boundary condition of zero vertical flux.

Q, the constant rate of emission of source is equal to $\int_0^x uCdz$ and C→∞ at (x=0=z).

Much work has been done on obtaining analytical solutions for K(z) and u. These solutions should be applied to ground-level sources only, for which eddy sizes are generally less than plume size. Reborts (1923) obtained a solution to equation (3) with wind varying as power of the height and eddy diffusivity varying as a power of height.

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The effective height:

The plume height Δh of diffusing matter is the distance from the stack height h_s to the point at the edge of the plume. The plume height has been calculated adopting the following equation (IAEA Safety Guide (1983)).

$$\Delta h = 3(w/u)D_1 \tag{4}$$

Where w is the exit velocity of the pollutants, and D_1 is the internal stack diameter. The effective stack height H equals:

$$H = h_s + \Delta h = h_s 3(w/u)D_1 \tag{5}$$

5- Concentration Profile:

The existing theories of diffusion assume or specify the concentration profile in the vertical. It is very difficult to verify a concentration profile because of the practical difficulties in measuring concentration with sufficient accuracy in the atmospheric layer of interest. The atmospheric layer being 300-500m thick, there is just not enough reliable data obtained in controlled experiments on diffusion studies.

The concentration profile is assumed in the form:

$$C/C_0 = 1 + \alpha_1 (z/H) + \alpha_2 (z/H)^2 + ... \tag{6}$$

C_o is concentration at the axis of the plume..
C is the concentration at distance z away from the axis of the plume.
H is the height of the plume, and $\alpha 1$ and α_2, etc are constants.

The number of terms chosen in the above series will depend upon desired goodness of fit to the observed data as shown in figure (2). It was found that the series in equation (6) gives a fairly good fit to the observed data even if only the first two terms are retained; that is,

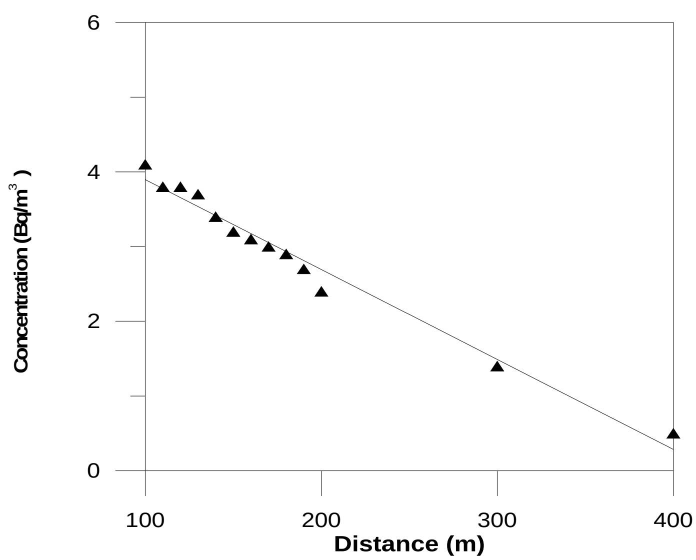

Fig. 2: Variation of the concentration of Iodine (I131) with distance from the reactor. Solid line is a straight line fit.

$$C/C_0 = 1 + \alpha_1 (z/H) \tag{7}$$

The above equation is for a straight line. The value α_1 will depend upon the concentration desired at the edge of the plume. If the edge of the plume is defined as having r per cent of the concentration, then

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α_1= -1+ 0.01 r (8a)

And if r = 0 then α_1= -1

C/C_o=1-(z/H) (8b)

6-Proposed Model:

The model proposed here is meant for use in connection with conservative material. The components of the conservation of mass, equation (1), namely wind profile u(z), effective height H and concentration profile C(z) have been discussed in detail in sections 3-5 respectively. Substituting for u(z) from equations (2), H from equation (5) and C(z) from equation (6) and using table 1 for n in different cases of stabilities. We get

$$Q = \int_0^H u_1 \left(\frac{z}{10}\right)^n C_0 \left[1 + \alpha\left(\frac{z}{H}\right)\right] dz \tag{9}$$

which after integrating; yields:

$$Q = \frac{u_1 C_0}{10^n} H^{n+1} \left(\frac{1}{n+1} + \frac{\alpha}{n+2}\right) \tag{10}$$

If α = -1 at r = 0, then

$$\frac{C_0}{Q} = \frac{10^n}{u_1 H^{n+1}} \left(\frac{1}{n+1} - \frac{1}{n+2}\right)^{-1} \tag{11a}$$

$$\frac{C_0}{Q} = \frac{\beta}{u_1 H^{n+1}} \tag{11b}$$

Where $\beta = 10^n \left(\frac{1}{n+1} - \frac{1}{n+2}\right)^{-1}$

where Co is the concentration at the edge of the plume axis at a place where the plume effective height is H. Q is the strength of the point source. β is a constant has different values during different stabilities given in Table 2.

Table 2: Values of constant β for different cases of stabilities

	Stability case		
	Stable	Neutral	Unstable
β	11.86	4.18	3.76

7-Case study:

It is useful to apply the derived expression for Co/Q on the first research reactor at Inshas. A continuous Ventilation system is provided with the reactor to the areas where radioactive gases, volatile materials and suspended particles can exist due to either leakage or airborne radioactivity. The total ventilation rate which could be emitted from the reactor stack of 43 m height, 1 m internal diameter, and exist velocity 4 m/s is 39965 m^3/hr, (Report 53 of Reactor Physics Department). α= -1 at r=0

The calculated values of U, Δh, H and Co/Q of neutral, stable and unstable conditions according are presented in Table(3), (4) and (5) respectively. The last columns in the three tables are given in 48 hours that are the usual continuous operation time of the reactor. The all values in the last column in tables (3), (4) and (5) (the concentration at the axis of the plume at the reactor release over emission rate) are inversely proportionality with the values of the wind and the effective heights. This means that if the values of the mean wind speed and the effective height are increasing, the values of the last column are decreasing.

Figure (2) shows that a straight line fits well to this data in the case neutral, stable and unstable conditions between the concentration at the plume axis over emission rate Co/Q and the effective height H.

Table 3: Wind speed, the plume rise, effective height and the concentration at the axis of the plume at the reactor release over emission rate during the year 1999 in neutral classes.

U (m/s)	Δh(m)	H (m)	$C_o/Q *10^3$ sec/m3
5.27	2.28	45.28	8.17
5.31	2.26	45.26	8.11
5.34	2.25	45.25	8.07
6.37	1.88	44.88	8.83
5.17	2.32	45.32	8.32
4.45	2.70	45.70	9.57
5.1	2.35	45.35	8.43
4.81	2.49	45.49	8.9
5.3	2.26	45.26	8.13
4.86	2.47	45.47	8.8
5.36	2.24	45.24	8.04
5.19	2.31	45.31	8.29
5.41	2.22	45.22	7.97
5.54	2.17	45.17	7.79
5.2	2.31	45.31	7.27
5.61	2.14	45.14	7.7
5.79	2.07	45.07	7.48
6.27	1.91	44.91	6.94
5.93	2.02	45.02	7.31
6.01	2.00	45.00	7.22
5.41	2.22	45.22	7.97
5.75	2.09	45.09	7.53
5.26	2.28	45.28	8.19

Table 4: Wind speed, the plume rise, effective height and the concentration at the axis of the plume at the reactor release over emission rate during the year 1999 in stable classes.

u (m/s)	Δh(m)	H (m)	$C_o/Q *10^3$ sec/m^3
4.43	2.71	45.71	8.7
3.81	3.15	46.15	9.93
4	3.00	46.00	9.5
4.92	2.44	45.44	7.87
3.7	3.24	46.24	10.19
3.57	3.36	46.36	10.52
3.64	3.30	46.30	10.34
3.45	3.48	46.48	10.85
3.6	3.33	46.33	10.45
3.8	3.16	46.16	9.95
3.99	3.01	46.01	9.52
3.89	3.08	46.08	9.75
3.75	3.20	46.20	10.07
3.98	3.02	46.02	9.54
3.47	3.46	46.46	10.79
4.06	2.96	45.96	9.37
4.3	2.79	45.79	8.9
4.31	2.78	45.78	8.88
4.02	2.99	45.99	9.46
4.11	2.92	45.92	9.27
3.94	3.05	46.05	9.63
3.86	3.11	46.11	9.81
3.67	3.27	46.27	10.27

Table 5: Wind speed, the plume rise, effective height and the concentration at the axis of the plume at the reactor release over emission rate during the year 1999 in unstable classes.

U (m/s)	Δh(m)	H (m)	$C_o/Q *10^6$ sec/m^3
4.43	2.71	45.71	9.68
3.81	3.15	46.15	11.13
4	3.00	46.00	10.65
4.92	2.44	45.44	8.78
3.7	3.24	46.24	11.44
3.57	3.36	46.36	11.82
3.64	3.30	46.30	11.61
3.45	3.48	46.48	12.19
3.6	3.33	46.33	11.73
3.8	3.16	46.16	11.16

3.99	3.01	46.01	10.67
3.89	3.08	46.08	10.1
3.75	3.20	46.20	11.3
3.98	3.02	46.02	10.69
3.47	3.46	46.46	12.13
4.06	2.96	45.96	10.24
4.3	2.79	45.79	9.96
4.31	2.78	45.78	9.94
4.02	2.99	45.99	10.6
4.11	2.92	45.92	10.38
3.94	3.05	46.05	10.79
3.86	3.11	46.11	11.0
3.67	3.27	46.27	11.52

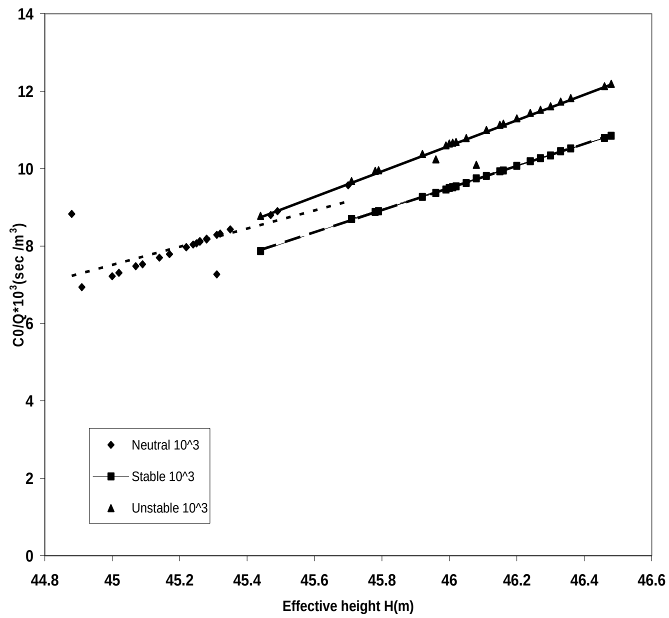

Fig. 2: Variation of the concentration at the plume axis over emission rate with the effective height H in neutral, stable and unstable conditions.

Verification:

For a point source locate at height h_s=27m (height of the source of the Second Research Reactor in A.E.A, Egypt (ETRR-2) from the ground. For Iodine (I_{138}), the height of the plume (H) is 31.29m, the total material discharge per unit second (Q) is 35 Bq, the wind speed (u_1) is 2.8 m/s and the lapse rate ($\Delta T/\Delta Z$) (C°/100m) is 0.36. This is the case of stable stratification (n=0.5).Using eq. (11a), we get the concentration at the axis of the plume (C_0) equal 0.847 Bq/m^3. Then the concentration at ground modifies to

$$C(ground) = 0.847(1-\frac{H_1}{H}) = 0.12\,Bq/m^3 \quad (12)$$

The observed concentration at a distance X=300m (with corresponding H=31.29 m) was 0.16 Bq/m^3. For the purpose of verification, they recommend that the source strength be adjusted to yield observed concentration at the first point of observation. For the test in question, the source strength is corrected by

$$Q(corrected) = \frac{0.16\times 35}{0.12} = 46.7\,Bq$$

By using Q (corrected), we get C (ground) =0.155 Bq/m^3.

9- Summary and conclusions:

The model presented described the pollutant concentrations downwind of a point source emitting pollutants into the atmosphere, the mean wind direction being located normal to the point source. The results of this study demonstrate:

1. The power law for wind profile is a good approximation for the planetary boundary layer. In this study The power law suggested U.S. Environmental Protection Agency (EPA) models has been used to get three different formulas for power law (neutral, stable and unstable classes)., the model is being extended of the area and line source configurations.
2. We calculate the plume rise, effective height and the concentration at the axis of the reactor release over emission rate through different stability classes.
3. Also we calculate the concentration at the ground of the Iodine (I_{138}) which agree with the observed concentration value after corrected its source strength.

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