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Studying Convergence and Stability of Fuzzy Differential Equation with Predictor Three-step and Nyström Method

Maryam Zohour Attar

3Sama Technical and Vocational Departmanet, Islamic Azad University, Dezful Branch, Dezful, Iran.

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ABSTRACT

Differential equations have many applications in different branches of sciences and engineering and play essential role in different fields such as mathematics, physics, statistics, engineering and social sciences. We want to solve differential equations with fuzzy method. Many scientific and engineering problems need to solve fuzzy differential equation (FDE) and each one of them holds true in initial fuzzy conditions. In this paper, some numerical methods are studied to solve the first order initial value problem with predictor three-step method based on Adams-Bashforth Predictor Method and Adams- Moulton corrector method and Nyström Method.The enough conditions for stability and convergence of the mentioned algorithms have been mentioned and studied by two examples and by drawing the diagram with MATLAB software. Results of this paper show that the predictor three-step method has higher convergence than the Nyström Method.

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INTRODUCTION

This paper studies subjects of the fuzzy differential equations (FDEs) and fuzzy integral equations (FIEs) which have grown rapidly in recent years. Marking of fuzzy differential equations was first introduced by Kandel [14]Byatt [11] and then applied in fuzzy processes and fuzzy dynamic systems in [15,16]. The complete theoretical researches of the first order fuzzy initial value problem were conducted by [8] Kaleva,[19] Seikkala,[17] Kloeden and [20][3] Baidosov, ‘ ‹liland], [8], Colombo[4] and Krivan[7]. Different programs of FDE were introduced in fuzzy control [8,9] which have been presented in some applications of numerical methods such as Eulerfuzzy method , Adams-Bashforth, Adams – Moulton and prediction and correction in FDE[1,2]. In this paper, fuzzy primary definitions have been first presented briefly and then fuzzy differential equations have been introduced with Adams-Bashforth, Adams – Moulton, predictor three-step method and Nyström Method, convergence and stability of these methods have been mentioned. The proposed algorithms have been shown by solving an example for both methods.

1- Definitions:

2-1- Differential Equation:

M-step method can be shown as the following equation to solve the initial value problem relating to a differential equation to find approximation of $y(t_{i+1})$ in t_{i+1} network points.

$$y(t_{i+1}) = a_{m-1}y(t_i) + a_{m-2}y(t_i - 1) + \dots + a_0 y(t_{i+1-m}) + h\{b_m f(t_{i+1}, y_{i+1}) + b_{m-1} f(t_i, y_i) + \dots + b_o f(t_{i+1-m}, y_{i+1-m})\}, \quad (1\text{-}1)$$

for $i = m-1, m, \dots, n-1$ so that $a = t_0 \le t_1 \le \dots \le t_n = b$

For ginen constants $b_m, \dots, b_1, b_0, a_{m-1}, \dots, a_1, a_0, h = \dfrac{(b-a)}{n} = t_{i+1} - t_i$ With initial values

$$y_0 = \alpha_0, y_1 = \alpha_1, y_2 = \alpha_2, \dots, y_{m-1} = \alpha_{m-1}$$

Corresponding Author: Maryam Zohour Attar, 3Sama Technical and Vocational Departmanet, Islamic Azad University, Dezful Branch, Dezful, Iran.
E-mail: M.zohuratar@gmail.com

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When b_m=0is called express method, it is obtained from expres(1-1)$y_{i+1}$of which values were determined in the previous sentences. When $b_m \neq 0$, it is called implicit method because y_{i+1} is on two sides of Equation (1-1) and is specified implicitly. Considering definition [2], we refer to some multiple step method as follows:

A- *Two-step Adams-Moulton method:*

$$y_0 = \alpha_0 \quad , \quad y_1 = \alpha_1$$

$$y_{i+1} = y_i + \frac{h}{12}[5f(t_{i+1}, y_{i+1}) + 8f(t_i, y_i) - f(t_{i-1}, y_{i-1})], \forall i = 1,2,..., n-1$$

B- *Three-step Adams- Bashforth method:*

$$y_0 = \alpha_0 \quad , \quad y_1 = \alpha_1 \quad , y_2 = \alpha_2$$

$$y_{i+1} = y_i + \frac{h}{12}[23f(t_i, y_i) - 16f(t_{i-1}, y_{i-1}) + 5f(t_{i-2}, y_{i-2})], \forall i = 2,3,..., n-1$$

2-2- Definition of Differential equation:

$$y_{i+1} = a_{m-1}y_i + a_{m-2}y_{i-1} + \ldots + a_0 y_{i+1-m} + ihF(t_i, h, y_{i+1}, y_i, \ldots, y_{i+1-m})$$

$$y_0 = \alpha_0 \quad , \quad y_1 = \alpha_1 \quad , \ldots\ldots, \; y_{m-1} = \alpha_{m-1} \tag{2-1}$$

The characteristic polynomial is in the opposite line: $P(\lambda) = \lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2} - \ldots - a_1\lambda - a_0$

If $|\lambda_i| \leq 1$ for each $i = 1,2...,m$, all roots are simple roots with absolute value of 1. Therefore, conditions of the root hold true in the differential method.

2-3- Theorem: a multiple-step method (2-1) is stable if and only if conditions of the root hold true[7].

2-4- Definition: fuzzy integral of $\int_a^b y(t)dt, \quad 0 \leq a \leq b \leq 1$ is defined as follows:

$$\left[\int_a^b y(t)dt\right]^\alpha = [\int_a^b \underline{y}^\alpha(t)dt, \int_a^b \overline{y}^\alpha(t)dt]$$

Provided that there is accurate Lebesgue Integral.

2-5- Note: if $y: I \to E$ is differentiable and y' is derivative on [0,1] which integrable, then:

$$y(t) = y(t_0) + \int_{t_0}^{t} y'(s)ds$$

For each value t,$t_0$where $t_0, t \in I$

2-6- Theorem: let $i = 0,1,...,n$,$(t_i, \tilde{u}_i)$ be the calculated data given that each $\tilde{u}_i = (\tilde{u}_i^l, \tilde{u}_i^c, \tilde{u}_i^r)$ is a member of E, then, for each $t \in [t_0, t_n]$ and $\tilde{f}(t) = (f^l(t), f^c(t), f^r(t)) \in E$

$$f^l(t) = \sum_{l_i(t) \geq 0} l_i(t) u_i^l + \sum_{l_i(t) \prec 0} l_i(t) u_i^r$$

$$f^c(t) = \sum_{i=0}^{n} l_i(t) u_i^c$$

$$f^r(t) = \sum_{l_i(t) \geq 0} l_i(t) u_i^r + \sum_{l_i(t) \prec 0} l_i(t) u_i^l$$

$$l_i(t) = \Pi_{j \neq i}^{n} \frac{t - t_j}{t_i - t_j}$$

Proof: refer to [10].

C-Adams- Bashforth methods:

Based on [2], the following results are obtained from Adams- Bashforth method:

$$\underline{y}^{\alpha}(t_{i+2}) = \underline{y}^{\alpha}(t_{i+1}) + \frac{h}{12}[5\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1})) - 16\overline{f}^{\alpha}(t_i, y(t_i)) + 23[\underline{f}^{\alpha}(t_{i+1}, y(t_{i+1}))]$$

$$\overline{y}^{\alpha}(t_{i+2})] = \overline{y}^{\alpha}(t_{i+1}) + \frac{h}{12}[5\overline{f}^{\alpha}(t_{i-1}, y(t_{i-1}) - 16\underline{f}^{\alpha}(t_i, y(t_i)) + 23\overline{f}^{\alpha}(t_{i+1}, y(t_{i+1}))]$$

$$\underline{y}^{\alpha}(t_{i-1}) = \alpha_0, \underline{y}^{\alpha}(t_i) = \alpha_1, \underline{y}^{\alpha}(t_{i+1}) = \alpha_2, \overline{y}^{\alpha}(t_{i-1}) = \alpha_3, \overline{y}^{\alpha}(t_i) = \alpha_4, \overline{y}^{\alpha}(t_{i+1}) = \alpha_5$$

D-Adams-Moulton methods:

Based on [2], we show two-step Adams-Moulton method as follows:

$$\underline{y}^{\alpha}(t_{i+1}) = \underline{y}^{\alpha}(t_i) - \frac{h}{12}[\overline{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + \frac{2h}{3}\underline{f}^{\alpha}(t_i, y(t_i)) + \frac{5h}{12}\underline{f}^{\alpha}(t_{i+1}, y(t_{i+1})),$$

$$\overline{y}^{\alpha}(t_{i+1}) = \overline{y}^{\alpha}(t_i) - \frac{h}{12}[\underline{f}^{\alpha}(t_{i-1}, y(t_{i-1})) + \frac{2h}{3}\overline{f}^{\alpha}(t_i, y(t_i)) + \frac{5h}{12}\overline{f}^{\alpha}(t_{i+1}, y(t_{i+1}))$$

$$\underline{y}^{\alpha}(t_{i-1}) = \alpha_0, \underline{y}(t_i) = \alpha_1 \overline{y}^{\alpha}(t_{i-1}) = \alpha_2, \overline{y}^{\alpha}(t_i) = \alpha_3$$

E-Three-step predictor method:

The following algorithm is presented based on three-step Adams- Bashforth method which predicts it once and corrects it with two-step Adams-Moulton method. The algorithm (Three-step predictor method) for approximate solution of the fuzzy initial value problem is as follows:
N is selected as positive integer:

$$\begin{cases} y'(t) = f(t, y(t)), t_0 \le t \le T \\ \underline{y}^{\alpha}(t_0) = \alpha_0,\ \underline{y}^{\alpha}(t_1) = \alpha_1,\quad \underline{y}^{\alpha}(t_2) = \alpha_2 \\ \overline{y}^{\alpha}(t_0) = \alpha_3, \overline{y}^{\alpha}(t_1) = \alpha_4,\quad \overline{y}^{\alpha}(t_2) = \alpha_5 \end{cases}$$

Step 1: We assume that:

$$h = \left(\frac{T - t_0}{n}\right) \begin{matrix} \underline{w}^{\alpha}(t_0) = \alpha_0, \underline{w}^{\alpha}(t_1) = \alpha_1, & \underline{w}^{\alpha}(t_2) = \alpha_2 \\ \overline{w}^{\alpha}(t_0) = \alpha_3, \overline{w}^{\alpha}(t_1) = \alpha_4, & \overline{w}^{\alpha}(t_2) = \alpha_5 \end{matrix}$$

Step 2: We assume thati=1

Step 3: we assume that

$$\begin{cases} \underline{w}^{(0)\alpha}(t_{i+2}) = \underline{w}^{\alpha}(t_{i+1}) + \dfrac{h}{12}[5\underline{f}^{\alpha}(t_{i-1}, \underline{w}(t_{i-1})) - 16\overline{f}^{\alpha}(t_i, w(t_i)) + 23\underline{f}^{\alpha}(t_{i+1}, w(t_{i+1}))] \\ \overline{w}^{(0)\alpha}(t_{i+2}) = \overline{w}^{\alpha}(t_{i+1}) + \dfrac{h}{12}[5\overline{f}^{\alpha}(t_{i-1}, \underline{w}(t_{i-1})) - 16\underline{f}^{\alpha}(t_i, w(t_i)) + 23\overline{f}^{\alpha}(t_{i+1}, w(t_{i+1}))] \end{cases}$$

Step 4- we assume that $t_{i+2} = t_0 + (i+2)h$

Step 5- We assume that:

$$\underline{w}^{\alpha}(t_{i+2}) = \underline{w}^{\alpha}(t_{i+1}) - \frac{h}{12}\overline{f}^{\alpha}(t_i, \underline{w}(t_i)) + (\frac{2h}{3})\underline{f}^{\alpha}(t_{i+1}, w(t_{i+1})) + (\frac{5h}{12})\underline{f}^{\alpha}(t_{i+2}, w(t_{i+2}))$$

$$\overline{w}^{\alpha}(t_{i+2}) = \overline{w}^{\alpha}(t_{i+1}) - \frac{h}{12}\underline{f}^{\alpha}(t_i, \overline{w}(t_i)) + (\frac{2h}{3})\overline{f}^{\alpha}(t_{i+1}, w(t_{i+1})) + (\frac{5h}{12})\overline{f}^{\alpha}(t_{i+2}, w(t_{i+2}))$$

Step 6: i=i+1

Step 7: If $i \le N - 2$, go to step 3.

Step 8: the algorithm is complete. $(\underline{Y}^{\alpha}(T), \overline{Y}^{\alpha}(T))$ is an accurate value of approximation $(\underline{w}^{\alpha}(T), \overline{w}^{\alpha}(T))$.
Step 3 predicts it with Adams- Bashforth method and step 5 corrects it with two-step Adams-Moulton method.

3-Convergence and Stability

We integrate two-step Adams-Moulton method from t_0 with a prefix $> t_0$, interval $[t_0,T]$ will be replaced with a set of separate network points space of $t_0 < t_1 < ... < t_n = T$ and $(\underline{Y}(t,\alpha), \overline{Y}(t,\alpha))$ is accurate solution and $(\underline{y}(t,\alpha), \overline{y}(t,\alpha))$ is approximate solution.

Accurate and approximate solutions t_n ، $0 \le n \le N$ are obtained with $Y_n(\alpha) = [\underline{Y}_n(\alpha), \overline{Y}_n(\alpha)]$ and $y_n(\alpha) = [\underline{y}_n(\alpha), \overline{y}_n(\alpha)]$, respectively. Each one of the network points can be calculated as $1 \le n \le N$, $h = (T - t_0)/N$, $t_n = t_0 + nh$.

From polygonal curvature two-step Adams-Moulton Equation:

$$\underline{y}(t,h,\alpha) = \{[t_0, \underline{y}_0(\alpha)], [t_1, \underline{y}_1(\alpha)], \ldots, [t_N, \underline{y}_N(\alpha)]\},$$

$$\overline{y}(t,h,\alpha) = \{[t_0, \overline{y}_0(\alpha)], [t_1, \overline{y}_1(\alpha)], \ldots, [t_N, \overline{y}_N(\alpha)]\},$$

Adams-Moulton's approximations are $\overline{Y}(t,\alpha), \underline{Y}(t,\alpha)$ on interval $t_0 \le t \le t_N$.

The following methods are applied for showing convergence of these approximations. It means that:

$$\lim_{h\to 0} \underline{y}(t,h,\alpha) = \underline{Y}(t,\alpha) \quad , \quad \lim_{h\to 0} \overline{y}(t,h,\alpha) = \overline{Y}(t,\alpha)$$

3-1- Method: Assume that $\{w_n\}_{n=0}^{N}$ is a string of numbers which holds true as follows:

$$|w_{n+1}| \le A|w_n| + B|w_{n-1}| + c \qquad 0 \le n \le N-1$$

For some known and positive constants A, B, C. Then: n odd and n even as n

$$|w_n| \le (A^{n-1} + \beta_1 A^{n-3}B + \beta_2 A^{n-5}B^2 + \ldots + \beta_s B^{[\frac{n}{2}]})|w_1| + (A^{n-2}B + \gamma_1 A^{n-4}B_2 + \ldots + \gamma_t AB^{[\frac{n}{2}]})|w_0|$$
$$+ (A^{n-2} + A^{n-3} + \ldots + 1)C + (\delta_1 A^{n-4} + \delta_2 A^{n-5} + \delta_m A + 1)BC + (\xi_1 A^{n-6} + \xi_2 A^{n-7} + \ldots + \xi_l A + 1)B^2C + (\lambda_1 A^{n-8} + \lambda_2 A^{n-9} + \ldots,$$
$$+ \lambda_p A + 1)B^3C + \ldots$$

$$|w_n| \le (A^{n-1} + \beta_1 A^{n-3}B + \beta_2 A^{n-5}B^2 + \ldots + \beta_s B^{[\frac{n}{2}]})|w_1| + (A^{n-2}B + \gamma_1 A^{n-4}B^2 + \ldots + \gamma_t AB^{[\frac{n}{2}]})|w_0|$$
$$+ (A^{n-2} + A^{n-3} + \ldots + 1)C + (\delta_1 A^{n-4} + \delta_2 A^{n-5} + \delta_m A + 1)BC + (\xi_1 A^{n-6} + \xi_2 A^{n-7} + \ldots + \xi_l A + 1)B^2C + (\lambda_1 A^{n-8} + \lambda_2 A^{n-9} + \ldots,$$
$$+ \lambda_p A + 1)B^3C + \ldots$$

So that all $\lambda_p, \xi_l, \delta_m, \gamma_t, \beta_s, p, l, m, t, s$ are fixed.

3-2- Theorem: for desirable and fixed r, $0 \le r \le 1$ in approximate two-step Adams- Moulton convergent to accurate solutions $\underline{Y}(t,\alpha), \overline{Y}(t,\alpha)$ for:

$$\underline{Y}, \overline{Y} \in C^4[t_0, T]$$

Proof: we should show that: by applying real values in the following results, we will obtain:

$$\lim_{h\to 0} \underline{y}_N(\alpha) = \underline{Y}(T,\alpha) \quad , \quad \lim_{h\to 0} \overline{y}_N(\alpha) = \overline{Y}(T,\alpha)$$

$$\underline{Y}_{n+1}(\alpha) = \underline{Y}_n(\alpha) - \frac{h}{12} f(t_{n-1}, \overline{Y}_{n-1}(\alpha)) + \frac{2h}{3} f(t_n, \underline{Y}_n(\alpha)) + \frac{5h}{12} f(t_{n+1}, \underline{Y}_{n+1}(\alpha)) - \frac{1}{24} h^4 \underline{Y}'''(\xi_n),$$

$$\overline{Y}_{n+1}(\alpha) = \overline{Y}_n(\alpha) - \frac{h}{12} f(t_{n-1}, \underline{Y}_{n-1}(\alpha)) + \frac{2h}{3} f(t_n, \overline{Y}_n(\alpha)) + \frac{5h}{12} f(t_{n+1}, \overline{Y}_{n+1}(\alpha) - \frac{1}{24} h^4 \overline{Y}'''(\xi_n),$$

As a result, $t_n < \underline{\xi}_n, \overline{\xi}_n < t_{n+1}$ where

$$\underline{Y}_{n+1}(\alpha) - \underline{y}_{n+1}(\alpha) = Y_n(\alpha) - \underline{y}_n(\alpha) - \frac{h}{12}\{f(t_{n-1}, \overline{Y}_{n-1}(\alpha)) - f(t_{n-1}, \overline{y}_{n-1}(\alpha))\}$$
$$+ \frac{2h}{3}\{f(t_n, \underline{Y}_n(\alpha)) - f(t_n, \underline{y}_n(\alpha))\} + \frac{5h}{12}\{f(t_{n+1}, \underline{Y}_{n+1}(\alpha)) - f(t_{n+1}, \underline{y}_{n+1}(\alpha))\} - \frac{1}{24} h^4 \underline{Y}'''(\xi_n),$$

$$\overline{Y}_{n+1}(\alpha) - \overline{y}_{n+1}(\alpha) = \overline{Y}_n(\alpha) - \overline{y}_n(\alpha) - \frac{h}{12}\{f(t_{n-1}, \underline{Y}_{n-1}(\alpha)) - f(t_{n-1}, \underline{y}_{n-1}(\alpha))\} + \frac{2h}{3}\{f(t_n, \overline{Y}_n(\alpha))$$
$$- f(t_n, \overline{y}_n(\alpha))\} + \frac{5h}{12}\{f(t_{n+1}, \overline{Y}_{n+1}(\alpha)) - f(t_{n+1}, \overline{y}_{n+1}(\alpha))\} - \frac{1}{24} h^4 \overline{Y}'''(\xi_n),$$

Then, we put $w_n = \underline{Y}_n(\alpha) - \underline{y}_n(\alpha), v_n = \overline{Y}_n(\alpha) - \overline{y}_n(\alpha)$.

$$|w_{n+1}| \le \left(1 + \frac{2hl_1}{3}\right)|w_n| + \left(\frac{hl_2}{12}\right)|v_{n-1}| + \left(\frac{5hl_3}{12}\right)|w_{n+1}| + \frac{1}{24} h^4 \underline{M},$$

$$|v_{n+1}| \le \left(1 + \frac{2hl_4}{3}\right)|v_n| + \left(\frac{hl_5}{12}\right)|w_{n-1}| + \left(\frac{5hl_6}{12}\right)|v_{n+1}| + \frac{1}{24} h^4 \overline{M}$$

We put:

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$\underline{M} = \max_{0\le t\le T} |\underline{Y}'''(t,\alpha)|, \overline{M} = \max_{0\le t\le T} |\overline{Y}'''(t,\alpha)|$ where : $l = \max\{l_1, l_2, l_3, l_4, l_5, l_6\} \prec \frac{12}{5h}$

Then:

$$|w_{n+1}| \le \left(1+\frac{13hl}{12-5hl}\right)|w_n| + \left(\frac{hl}{12-5hl}\right)|v_{n-1}| + \left(\frac{1}{24-10hl}\right)h^4\underline{M},$$

$$|v_{n+1}| \le \left(1+\frac{13hl}{12-5hl}\right)|v_n| + \left(\frac{hl}{12-5hl}\right)|w_{n-1}| + \left(\frac{1}{24-10hl}\right)h^4\overline{M}$$

As a result, we put $w_0 = v_0 = 0, |u_n| = |w_n| + |v_n|$ and also $w_1 = v_1 = 0$ in method 6-1

$$|u_n| \le \frac{(1+\frac{13hl}{12-5hl})^{n-1}-1}{\frac{13hl}{12-5hl}} \times \frac{1}{24-10hl} h^4(\underline{M}+\overline{M}) +$$

$$\left\{\delta 1(1+\frac{13hl}{12-5hl})^{n-5} + \delta_2(1+\frac{13hl}{12-5hl})+1\right\} + \ldots + \delta_m[1+\frac{13hl}{12-5hl}+1]\left((\frac{hl}{12-5hl})^2 \frac{1}{24-10hl} h^4(\underline{M}+\overline{M})\right)$$

$$+\{\xi_1(1+\frac{13hl}{12-5hl})^{n-6} + \xi 2((1+\frac{13hl}{12-5hl})^{n-7} + \ldots + \delta_n[1+\frac{13hl}{12-5hl})+1]\}\left((\frac{hl}{12-5hl})^3 \frac{1}{24-10hl} h^4(\underline{M}+\overline{M})\right)+$$

$$\{\lambda_1(1+\frac{13hl}{12-5hl})^{n-8} + \lambda_2((1+\frac{13hl}{12-5hl})^{n-9} + \ldots + \lambda_p[1+\frac{13hl}{12-5hl})+1]\}\left((\frac{hl}{12-5hl})^4 \frac{1}{24-10hl} h^4(\underline{M}+\overline{M})\right)+\ldots$$

If $h \to 0$, then, $v_n, w_n \to 0$. Therefore, proof is complete.

3-3- Note: the above theorem shows that convergence is of $o(h^3)$ order.

3-4- Note: we simply show that convergence order of two-step Adams- Bashforth method is $O(h^2)$.

3-5- Theorem: two-step and three-step Adams- Bashforth methods are the stable methods.

Argument: for two-step Adams- Bashforth method, there is only one characteristic polynomial $p(\lambda) = \lambda^2 - \lambda$. Clearly, theorem 2-1 holds true in root conditions. Then, two step Adams- Bashforth method is stable.

For three step Adams- Bashforth method, there is only one characteristic polynomial $p(\lambda) = \lambda^3 - \lambda^2$ which holds true in root conditions. Therefore, this method is also stable.

3-6-Theorem: two-step and three-step Adams- Moulton methods are the stable methods.

The reason for selection of Adams- Bashforth method and Adams- Moulton methods in predictor technique is their stability.

Example: $\tilde{y}'(t)=-\tilde{y}(t)+t+1$,

(0)=(0.96+0.04α ,1.01-0.01α),ỹ

0.01)=(0.01+(0.985+0.015α)$e^{-0.01}$-(1-α)0.025$e^{0.01}$,(ỹ

0.01+(0985+0.015α)$e^{-0.01}$+(1-α)0.025$e^{0.01}$),

ỹ (0.02)=(0.02+(0.985+0.015α)$e^{-0.02}$-(1-α)0.025$e^{0.02}$, 0.02+(0.985+0.015α)$e^{-0.02}$+(1-α)0.025$e^{0.02}$).

In t=0.1, real value is obtained.

Ỹ(0.1,α)=(0.1+(0.985+0.015α)$e^{-0.1}$-(1-α)0.025$e^{0.1}$, 0.1+(0.985+0.015α)$e^{-0.1}$+(1-α)0.025$e^{0.1}$).

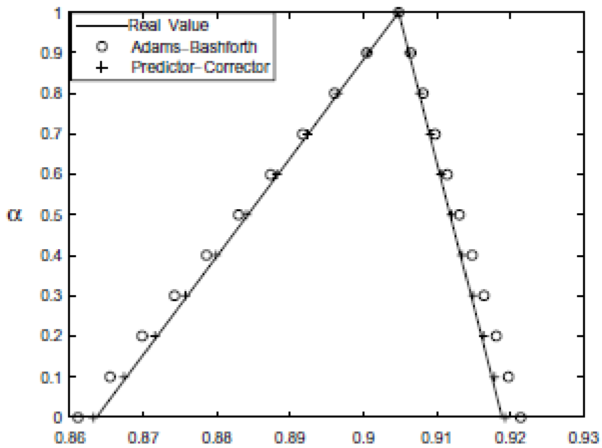

Fig. 1: Result of Implementing two-step Adams- Bashforth method and three-step predictor method

4-Nyström method:

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On special example of multiple step methods is Nystrom. Based on [5], we have a system as follows: $y_{i+1} = y_{i-1} + h\sum_{m=0}^{q} k_m \nabla^m f(t_i, y_i) \quad q = 0,1,2,...(4-1)$:where the constant value is equal to : $k^m = (-1)^m \int_{-1}^{1} \binom{-s}{m} ds$

Which is independent of f. $\nabla f(t_i, y_i), t = t_0 + sh$ is the first backward differencef(t,y(t)) in a point of t=t_i and backward differences are defined by $\nabla^k f(t_i, y_i) = \nabla(\nabla^{k-1} f(t_i, y_i))$.

Where q=0 is a special state of Nystrom method. We know that $y_{i+1}=y_{i-1}+2h\ f(t_i,y_i)$ is obtained from midpoint law.

Based on [16], $l_j(t)$ mark depends on q which can be even or odd. We assume that q is even. For q odd, we canprogress in this way. For $t_{i-1} \le t \le t_i$, we define $l_j(t)$. One can write:

$$l_k(t) \ge 0 \qquad for \qquad k \in M = \{0,1,3,...,q-1\}$$

$$l_k(t) < 0 \qquad for \qquad k \in N = \{2,4,...,q\}$$

And for $t_i \le t \le t_{i+1}$ $\begin{matrix} l_k(t) \ge 0 & for & k \in N' = \{0,2,4,...,q\} \\ l_k(t) < 0 & for & k \in M' = \{1,3,...,q-1\} \end{matrix}$

If we define $\delta_j = \int_{t_i}^{t_{i+1}} l_j(t)dt, \gamma_j = \int_{t_{i-1}}^{t_i} l_j(t)dt$, it is obtained from Nyström Method.

$$\begin{cases} \underline{y}_{i+1}^{\alpha} = \underline{y}_{i-1}^{\alpha} + \sum_{j\in M} \underline{f}^{\alpha}(t_{i-j}, y_{i-j})\gamma_j + \sum_{j\in N} \overline{f}^{\alpha}(t_{i-j}, y_{i-j})\gamma_j + \sum_{j\in N'} \underline{f}^{\alpha}(t_{i-j}, y_{i-j})\delta_j + \sum_{j\in M'} \overline{f}^{\alpha}(t_{i-j}, y_{i-j})\delta_j \\ \overline{y}_{i+1}^{\alpha} = \overline{y}_{i+1}^{\alpha} + \sum_{j\in M} \overline{f}^{\alpha}(t_{i-j}, y_{i-j})\gamma_j + \sum_{j\in N} \underline{f}^{\alpha}(t_{i-j}, y_{i-j})\gamma_j + \sum_{j\in N'} \overline{f}^{\alpha}(t_{i-j}, y_{i-j})\delta_j + \sum_{j\in M'} \underline{f}^{\alpha}(t_{i-j}, y_{i-j})\delta_j \\ \underline{y}_{i-q}^{\alpha} = \alpha_0, \underline{y}_{i-q+1}^{\alpha} = \alpha_1, \dots, \underline{y}_{i}^{\alpha} = \alpha_q \overline{y}_{i-q}^{\alpha} = \alpha_{q+1}, \overline{y}_{i-q+1}^{\alpha} = \alpha_{q+2}, \dots, \overline{y}_{i}^{\alpha} = \alpha_{2q+1} \end{cases}$$

In special case of q=0, it is better to write $\delta_0 = h \quad , \gamma_0 = h$. Therefore, the equation is converted as follows which is the midpoint law.

$$\begin{cases} \underline{y}_{i+1}^{\alpha} = \underline{y}_{i-1}^{\alpha} + 2h\underline{f}^{\alpha}(t_i, y_i) \\ \overline{y}_{i+1}^{\alpha} = \overline{y}_{i-1}^{\alpha} + 2h\overline{f}^{\alpha}(t_i, y_i) \\ \underline{y}_{i-1}^{\alpha} = \alpha_0, \underline{y}_{i}^{\alpha} = \alpha_1, \overline{y}_{i}^{\alpha} = \alpha_2, \overline{y}_{i}^{\alpha} = \alpha_3 \end{cases}$$

5- **Convergence and stability**: to complete the given system from t_o to prefix of $T > t_0$, $t_0 < t_1 < t_2 < ... < t_N = T$ should be replaced with a separate equal set of network point space in interval $[t_0,T]$. $(\underline{y}(t,\alpha), \overline{y}(t,\alpha))$ is the approximation of accurate solution of $(\underline{Y}(t_0,\alpha), \overline{Y}(t_0,\alpha))$. The accurate and approximate solution in t_n' is shown for $0 \le n \le N$ as follows: $[Y_n]^{\alpha} = [\underline{Y}_n^{\alpha}, \overline{Y}_n^{\alpha}] \quad , [y_n{}^{\alpha}] = [\underline{y}_n^{\alpha}, \overline{y}_n^{\alpha}]$

The network points are obtained with $0 \le n \le N, h = \frac{T-t_0}{N} \quad , t_n = t_0 + nh$ Relations. From [16], we have polygonal curve: $\begin{matrix} \underline{y}(t,h;\alpha) = \{[t_0, \underline{y}_0^{\alpha}], [t_1, \underline{y}_1^{\alpha}], \dots [t_N, \underline{y}_N^{\alpha}]\} \\ \overline{y}(t,h;\alpha) = \{[t_0, \overline{y}_0^{\alpha}], [t_1, \overline{y}_1^{\alpha}], \dots [t_N, \overline{y}_N^{\alpha}]\} \end{matrix}$

Nystrom approximations of $\overline{Y}(t.\alpha), \underline{Y}(t.\alpha)$ are on interval $t_0 \le t \le t_N$.

Based on the methods available in [16], we want to show convergence of approximations meaning that:

$$\lim_{h\to 0} \underline{y}(t,h;\alpha) = \underline{Y}(t;\alpha) \qquad , \lim_{h\to 0} \overline{y}(t,h;\alpha) = \overline{Y}(t;\alpha)$$

We show convergence and stability of the midpoint law that is q=0 and one can have similar proof for other values of q.

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We assume that G(t,u,v),F(t,u,v) are functions of G,F so that v,u are fixed and $u \le v$. In other words, G(t,u,v),F(t,u,v) are obtained by substituting in y=(u,v). The field in which G,F are defined is as follows:
$K = \{(t,u,v) \mid t_0 \le t \le T, -\infty < v < \infty, -\infty < u \le v\}$.

5-1- Theorem: let G(t,u,v),F(t,u,v) be dependent on C^1(K) if relative derivatives of G,F are bounded on K . Then, for fixed desirable α where $0 \le \alpha \le 1$ is approximation from midpoint convergent to accurate solution of $\overline{Y}(t,\alpha), \underline{Y}(t,\alpha)$ uniformly in t for $\underline{Y}, \overline{Y} \in C^3[t_0, T]$.

Argument: we should show that: for $\lim_{h\to 0} \underline{y}_N^{\alpha} = \underline{Y}(t;\alpha) \quad , \lim_{h\to 0} \overline{y}_N^{\alpha} = \overline{Y}(t;\alpha)$

Based on Taylor's theorem**:**
Where $t_{n-1} < \underline{\xi}n, \overline{\xi}n < t_{n+1}$ As a result:

$$\underline{Y}_{n+1}^{\alpha} - \underline{y}_{n+1}^{\alpha} = \underline{Y}_{n-1}^{\alpha} - \underline{y}_{n-1}^{\alpha} + 2h(F(t_n, \underline{Y}_n^{\alpha}, \overline{Y}_n^{\alpha}) - F(t_n, \underline{y}_n^{\alpha}, \overline{y}_n^{\alpha})\} + \frac{h^3}{3}\underline{Y}'''(\underline{\xi}n),$$

$$\overline{Y}_{n+1}^{\alpha} - \overline{y}_{n+1}^{\alpha} = \overline{Y}_{n-1}^{\alpha} - \overline{y}_{n-1}^{\alpha} + 2h(G(t_n, \underline{Y}_n^{\alpha}, \overline{Y}_n^{\alpha}) - G(t_n, \underline{y}_n^{\alpha}, \overline{y}_n^{\alpha})\} + \frac{h^3}{3}Y'''(\xi n),$$

Then, we put $v_n = \overline{Y}_n^{\alpha} - \overline{y}_n^{\alpha} \quad , w_n = \underline{Y}_n^{\alpha} - \underline{y}_n^{\alpha}$:

$$|w_{n+1}| \le |w_{n-1}| + \varepsilon hl \max\{|w_n|, |v_n|\} + \frac{h^3}{3}\underline{M}$$
$$|v_{n+1}| \le |v_{n-1}| + \varepsilon hl \max\{|w_n|, |v_n|\} + \frac{h^3}{3}\overline{M}$$

Where $\overline{M} = \max_{t_0 \le t \le T} |\overline{Y}'''(t;\alpha)|$, $\underline{M} = \max_{t_0 \le t \le T} |\underline{Y}'''(t;\alpha)|$ and $l > 0$ are a bound for relative derivatives of G,F. Therefore,

$$|w_{n+1}| + |v_{n+1}| \le |w_{n-1}| + |v_{n-1}| + 8hl \max\{|w_n|, |v_n|\} + \frac{h^3}{3}(\underline{M} + \overline{M}) \le |w_{n-1}| + |v_{n-1}| + 8hl\{|w_n| + |v_n|\} + \frac{h^3}{3}(\underline{M} + \overline{M})$$

If we put: $|u_n| = |w_n| + |v_n|$, then, $|u_{n+1}| \le 8hl|u_n| + |u_{n-1}| + \frac{h^3}{3}(\underline{M} + \overline{M})$. Therefore, $w_0 = v_0 = w_1 = v_1 = 0$ is obtained.

$$|u_n| \le \frac{(8hl)^{n-1} - 1}{8hl - 1} \times \frac{h^3}{3}(\underline{M} + \overline{M}) + \{\delta_1(8hl)^{n-4} + \delta_2(8hl)^{n-5} + \dots + \delta_m(8hl) + 1\} \times \frac{h^3}{3}(\underline{M} + \overline{M}) + \{\xi_1(8hl)^{n-6}$$
$$\xi_2(8hl)^{n-7} + \dots + \xi_1(8hl) + 1\}\frac{h^3}{3}(\underline{M} + \overline{M}) + \{\lambda_1(8hl)^{n-8} + \lambda_2(8hl)^{n-9} + \dots + \lambda_p(8hl) + 1\}\frac{h^3}{3}(\underline{M} + \overline{M}) + \dots$$

If $h \to 0$, then, $v_n \to 0 \quad , \quad w_n \to 0$ and the theorem is proved.
5-2- Result of the above theorem is convergence of o(h^2) order.
5-3- Midpoint law is stable.
Argument: there is characteristic polynomial point law $p(\xi) = \xi^2 - 1$. Clearly, root conditions are established. Therefore, the midpoint is stable.

6-Numerical results:

Now, we apply midpoint law for two points. We compare the approximate solutions with real solutions.
Example: Initial value problem conditions:
$\tilde{y}'(t) = -\tilde{y}(t) + t + 1$
$\tilde{y}(0) = (0.96 + 0.04\alpha, 1.01 - 0.01\alpha)$,
$\tilde{y}(0.01) = (0.01 + (0.985 + 0.015\alpha)e^{-0/01} - (1-\alpha)0.025e^{0/01}, 0.01 + (0.985 + 0.015\alpha)e^{-0/01} + (1-\alpha)0.025e^{0/01})$
Accurate solution in t=0.1 is as follows:
$Y(0.1.\alpha) = (0.1 + (0.985 + 0.015\alpha)e^{-0.1} - (1-\alpha)0.025e^{0.1} 0.1 + (0.985 + 0.015\alpha)e^{-0.1} + (1-\alpha)0.025e^{0.1})$,
$0 \le \alpha \le 1$
Its diagram is as follows:

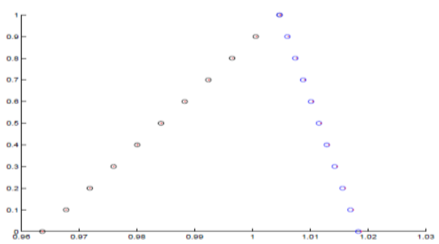

Fig. 2: Circles are approximate solutions and points are real solutions.

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7-Conclusion:

Based on [4], considering that convergence of Euler's method is of O(h) order , higher convergence was obtained with the methods mentioned in this Section. It means that the predictor method with $O(h^m)$ convergence order with m step Adams-Bashforth method and (m-1) step Adams- Moulton method were considered as predictor and corrector. Nystrom method is of) $O(h^2)$ convergence order. Therefore, three-step predictor method has higher convergence and the reason for selecting these methods for solving the fuzzy initial value problem is their stability.

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