Journal of Applied Science and Agriculture, 9(4) April 2014, Pages: 1371-1378

AENSI Journals

Journal of Applied Science and Agriculture

ISSN 1816-9112

Journal home page: www.aensiweb.com/jasa/index.html

Dynamics of Monthly Rainfall In Ramsar

[1]Rasoul Jani, [2]Mohammad Ali Ghorbani, [1]Abolfazl Shamsai

[1]*Department of Water Engineering, college of Technical and Engineering, Tehran Science and Research Branch, Islamic Azad University, Tehran, Iran.*
[2]*Department of Civil Engineering, Tabriz University, Tabriz, Iran.*

ARTICLE INFO

Article history:
Received 21 January, 2014
Received in Revised form 16 April, 2014
Accepted 22 April 2014
Available Online 5 May, 2014

Key words:
Chaos theory, Correlation dimension, Lyapunov exponent, Power spectrum, Ramsar

ABSTRACT

Background: Adequate knowledge of rainfall behavior is important for proper planning and management of water resources and design of environment. Chaotic theory, which is the basis and foundation of nonlinear dynamic systems, has caused a great revolution in how to understand and express phenomena. Chaos theory is a tool that can be used for chaotic nature and complexity and it can be appropriate for rainfall data with higher variation coefficient. **Objective:** In this study chaotic behavior of monthly rainfall in the Ramsar during January 1960 - December 2000 is investigated. A range of nonlinear dynamic methods have specifically been developed to identify chaotic behaviours mainly from time series. The methods and indicators of chaos theory (power spectrum, average mutual information, false nearest neighbours, correlation dimension and largest Lyapunov exponents) were applied. **Results:** The value of power spectrum (0.64) indicates that chaotic (fractal) behavior to the rainfall time series. The optimal delay time (4 month) and embedding dimension (13) are obtained from average mutual information and false nearest neighbours techniques, respectively. Optimal values are then used for the estimation of the correlation dimension and the largest Lyapunov exponent for inspecting possible signatures of chaotic dynamics. The low correlation dimension (2.82) suggests the presence of low-dimensional chaos; also imply that the rainfall dynamics are dominantly governed by three variables. The positive largest Lyapunov exponent value (0.0135) indicates a signature of chaos. **Conclusion:** These results give a positive indication towards considering rainfall as a chaotic system and it is predictably for 74 next months.

To Cite This Article: Rasoul Jani, Mohammad Ali Ghorbani, Abolfazl Shamsai., Dynamics of Monthly Rainfall In Ramsar. ***J. Appl. Sci. & Agric.,*** *9(4): 1371-1378, 2014*

INTRODUCTION

Temporal and spatial changes of rainfall, disorders in hydrological balances and management strategies. In recent decades, chaotic theory, which is the basis and foundation of nonlinear dynamic systems, has caused a great revolution in how to understand and express phenomena. This theory deals with studying the systems appearing irregular at first glance, however, they are in fact, ruled by determined laws. These systems are very sensitive to the initial conditions (Williams,1997; sivakumar,1999), in a way that the input which seems irregular is able to have a great effect on them. The systems like these are called chaotic systems.

Concerning the analysis of the rainfall process, some researchers have carried out some several studies in recent decades. Kyoung *et al*. (2011) indicated that rainfall data in Penisula located in Korea had optimal chaos with a suitable fractal dimension. A comparison of results from treating rainfall in the three regions by nagesh Kumar and dhanya (2011) in India indicates that although they are chaotic in nature, the spatial averaging over a large area can increase the dimension and improve the predictability. An effort based on the notion of deterministic chaos to investigate the behaviour of the dynamics of daily rainfall in Fujian province, China, is studied by zehua li *et al*.(2010). Results from the correlation dimension method and nonlinear prediction method, jointly indicate that the daily rainfall process exhibits nonlinear dynamics. However, different results from the past studies, the minimum number of variables essential is identified as 5 and the number of sufficient variables is up to 16. Temporal scale and variability and number of zeros in rainfall with chaotic behavior were investigated by sivakumar (2009), The result suggests that the low correlation dimensions for rainfall in particular finer-resolution ones that commonly have a large number of zeros. Still, other issues have been studied in relation to river-related time series, such data size (Sivakumar, 2005; Sivakumar *et al*., 2002), the application of chaotic analytical techniques to daily hydrologic series comprising of outliers, (Ng *et al*., 2007). Chaos models were investigated as a noise reduction technique by Porporato and Ridolfi (1997), Jayawardena and Gurung (2000); as well as a technique to estimate missing data by Elshorbagy *et al*. (2002a, b). SivaKumar

Corresponding Author: Mohammad Ali Ghorbani, Department of Civil Engineering, Tabriz University, Tabriz, Iran.

(2001) has studied the rainfall dynamics of Leaf River in Mississipi through chaotic theory in four different time stages. He has concluded that the data having higher Coefficient of Variation possess lower correlation dimension and vice versa.

Considering the importance of studying rainfall and the practicality of chaos theory in the analysis and prediction of this factor, Ramsar metropolis has been chosen as the case study. And the purpose of the present study is being aware of rainfall process with using chaotic signals. This paper aims at investigating the possible presence of chaotic signals in rainfall time series during January 1970- December 2000. The remainder of the paper is organized as follows. Section 2 presents the methodology in this study. In Section 3, the data used, case study and results obtained are explained. The conclusions of this study are presented in conclusion Section.

Methodology:

Power Spectrum Analysis:

Since chaotic systems are a periodic, a power spectrum analysis can indicate the presence of periodic regimes (Ng *et al*., 2007). If the power spectrum, E(f), obeys a power law form

$$E(f) \propto f^{-\beta} \tag{1}$$

where f is the frequency and β is the spectral exponent, this is an indication of the absence of characteristic time scale in the range of the power law. In such a case, fractal behaviour may be assumed to hold (Sivakumar,2006).

Phase space reconstruction:

The concept of phase space is a powerful tool for characterizing dynamical systems. The delay embedding is one of the most popular methods for reconstructing phase space from a univariate or multivariate time series (Takens,1981), assumed to be generated by a deterministic dynamical system with *D* degrees of freedom. The Takens theorem states that the underlying (unknown) dynamics can be fully recovered by building a *m*-dimensional space wherein the components of each state vector $\vec{Y}_t$ are defined through the delay coordinates

$$\vec{Y}_t = \left(X_t, X_{t-\tau}, X_{t-2\tau}, \ldots, X_{t-(m-1)\tau}\right) \tag{2}$$

where m > 2D2 is called embedding dimension and τ is referred to as delay time. If the dynamics of the system can be reduced to a set of deterministic laws, trajectories converge towards a subset of the phase space with fractional dimension, called attractor.

Many methods have been proposed for the estimation of optimal values of the embedding parameters: within this work we adopted the minimization of the False Nearest Neighbours (FNN) for m and of the Average Mutual Information (AMI) for τ , as suggested by Cellucci *et al*.(2003)

For a given time series sequence $\{x_0, x_1, x_2, ..., x_i, ..., x_n\}$ the mutual information indicates the amount of information about the state $x_{i+\tau}$ if the state of x_i is known. The average mutual information is defined by:

$$I(\tau) = -\sum_{ij}^{N} p_{ij}(\tau) \ln \frac{p_{ij}(\tau)}{p_i(\tau) p_j(\tau)} \tag{3}$$

Where $p_i(\tau)$ and $p_j(\tau)$ correspond to the probability of finding x_i in the ith and $x_{i+\tau}$ in the jth interval, and $p_{ij}(\tau)$ is their joint probability. The first local minimum of $I(\tau)$ estimates the optimal selection for the delay time required for phase space reconstruction. In practice, it provides the maximum delay time such that $x_{i+\tau}$ adds the largest amount of information about x_i .

Successively, false nearest neighbors (FNN) search is used to determine the optimal embedding dimension m. In fact, a small value of m may not be sufficient to reconstruct the phase space, whereas a large value of m causes a large unfolding of the attractor and high computational cost (Cellucci *et al*., 2003). The method is as follows. For a fixed embedding dimension m, and for each point in the phase space, we identify the K nearest neighbors. Then, we repeat our procedure in the m+1-dimensional phase space: if the reconstruction is not optimal, the nearest neighbors are different, i.e. they were false nearest neighbors.

FNN method employs the search of false neighbours in phase space: when the ratio between the number of false neighbours at the dimension m+1 and m is below a given threshold, generally smaller than 5%, each m' > m +1 is an optimal embedding. However, if m' is too large, a poor reconstruction of few embedding states with several components is obtained and the next analyses should not be performed (Kennel *et al*.,1992).

Correlation dimension:

Correlation dimension is the most widely used as fractal dimension quantifier, and is based on the correlation integral (Grassberger, Procaccia, 1998).

For an m-dimensional phase space, the correlation function $C_m(r)$ is defined as the fraction of states closer than r (Fraser,Swinney,1986):

$$C_m(r) = \lim_{N\to\infty} \frac{2}{(N-w)(N-w-1)} \sum_{i=1}^{N} \sum_{j=i+1+w}^{N} H(r - |\vec{Y}_i - \vec{Y}_j|) \quad (4)$$

Where H is the Heaviside step function, $\vec{Y}_i$ is the *i*-th state vector, N is the number of points on the reconstructed attractor and r is the radius of the sphere centered on Y_i or Y_j.

The number w is called *Theiler window* and it is the correction needed to avoid spurious results due to temporal correlations instead of dynamical ones. For stochastic time series $C_m(r) \propto r^m$ holds, whereas for chaotic time series the correlation function scales with r as:

$$C_m(r) \propto r^{D_2} \quad (5)$$

where D_2, called *correlation exponent.* The correlation exponent is defined by

$$D_2 = \lim_{r\to 0} \frac{\ln C_m(r)}{\ln r} \quad (6)$$

And can be reliably estimated as the slope in the $\ln C_m(r)$ vs. $\ln(r)$ plot. The slope can be computed by the least-squares fit of a straight line over a length scales of r.

According to Grassberger-Procassia algorithm (1983), in case of deterministic data set the plot of 'm' versus 'D_2' should be a straight line parallel to embedding dimension, in case of stochastic data set, it should be straight line sloping 45 degrees to x and y axis . For chaotic system, the correlation exponent initially increases but finally saturates after an especial embedding dimension. The saturation value of the correlation exponent is defined as the correlation dimension. Fig.1 shows the above three cases (Stochastic, Deterministic, Chaotic) characteristics.

If the value of correlation dimension is small and fractal, in this case the system is a low-dimensional deterministic chaotic dynamic.

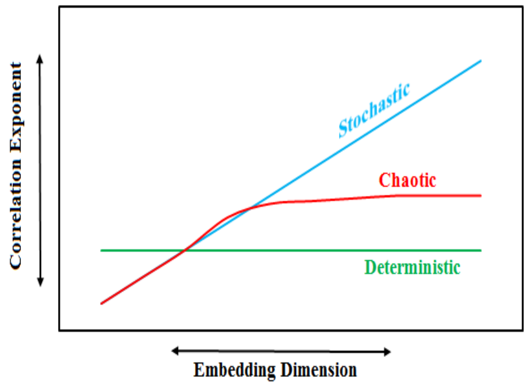

Fig. 1: Plot differentiating deterministic, stochastic and chaotic system.

Lyapunov exponents:

The unpredictability is one of the important characteristics of a chaotic system, because the sensitive dependence on initial conditions. The largest Lyapunov exponent need only be considered, as it determines the total predictability of the system (Kumar, Dhanya, 2011). In general, Lyapunov exponent (λ) is a quantitative measure of the sensitive dependence on the initial conditions and to discriminate between chaotic dynamics and periodic signals are often used. Lyapunov exponents quantify the divergence of nearby trajectories in the phase space, along a given direction. Given two nearby states and their Euclidean distance $d(t_0)$ at time t_0, the largest Lyapunov exponent λ_{max}, corresponding to the dominant divergence direction, is defined as:

$$\lambda_{max} = \lim_{t\to\infty} \frac{1}{t - t_0} \log \frac{d(t)}{d(t_0)} \quad (7)$$

In the present work we adopt the method proposed by Rosenstein *et al.*(1993). For the estimation of λ_{max}: it makes use of the stretching factor

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$$S(t) = \frac{\Delta t}{t} \sum_{i=1}^{t/\Delta t} \log\left[\frac{1}{|\Omega_i|} \sum_{j \in \Omega_i}^{N} \left| \vec{Y}_i - \vec{Y}_j \right| \right] \quad (8)$$

Along an orbit of $\Delta t / t$ time steps, where $|\Omega_i|$ is the number of neighbors in the neighborhood Ω_i of the reference state $\vec{Y}_i$, and Δt is the sampling time of measurements. For a chaotic dynamics, the stretching factor $S(t)$ is expected to be proportional to time, the largest Lyapunov exponent $\lambda_{\max}$ being the proportionality constant. In other word in the case of chaotic dynamics, a plot of the stretching factor S against the number of points N (or time $t = N \cdot \Delta t$) will yield a curve with a linear increase at the beginning, followed by an almost flat region. The slope of the linear portion of the first part of this curve gives an estimate of $\lambda_{\max}$ (Rosenstein *et al*., 1993).To be chaotic, $\lambda_{\max}$ must exceed zero. Only for systems with $\lambda_{\max}$ between zero and one are chaotic predictions of any practical use. Usually in practice, one is interested in the maximal Layapunov exponent that can be used to categories the type of the motion of the system as presented in Table 1.

Table 1: Possible types of dynamics systems and the corresponding maximum lyapunov exponents (Siek 2011).

Maximum Lyapunov exponent	$\lambda_{\max} \prec 0$	$\lambda_{\max} = 0$	$0 \prec \lambda_{\max} \prec \infty$	$\lambda_{\max} = \infty$
Type of dynamics system	Stable fixed point	Stable Limit cycle	Deterministic chaos	Noise (Random motion)

Area and the data used:

The monthly rainfall data during January 1970 - December 2000 of Ramsar, Ramsar city is located in the state of Mazandaran on the coast of Caspian Sea. The city of Ramsar borders the Caspian Sea to the north (Fig. 2). The city has nearly 688 square kilometers area composed of two distinct sections: Mountainous and low valleys. The bedrock of this terrain is sandstone, covered with dense forests and the annual precipitation is 1200 mm.

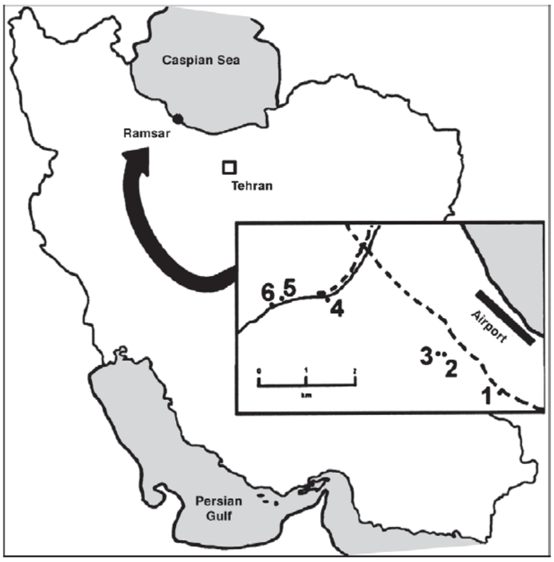

Fig. 2: The location of Ramsar in Iran.

The statistical parameters of rainfall data are given in Table 2 and Fig.3 shows the variations of monthly rainfall data series.

Table 2: Statistics of monthly rainfall time series at the Ramsar.

Statistic	Value
Number of data	488
Mean(m)	94.19
Standard deviation(m)	70.83
Maximum value(m)	426.39
Minimum value(m)	16.35
Variance(m2)	5105.5
Skewness	2.00
Kurtosis	3.87

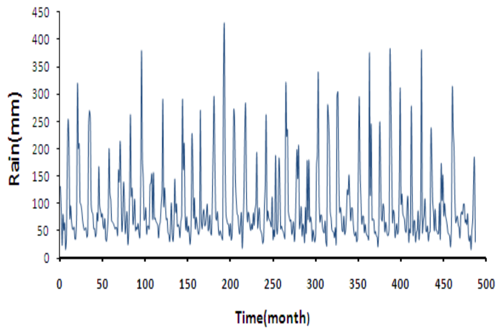

Fig. 3: Monthly Rainfall time series (January 1960 - December 2000).

RESULTS AND DISCUSSIONS

Power Spectrum:

Figure 4 shows the power spectrum of the monthly rainfall series observed in the Ramsar. The spectrum has been averaged over logarithmically spaced frequency intervals. The value of the β computed from the slope of the solid line, is approximately 0.64, and this an indication that chaotic (fractal) behavior to rainfall time series during this time interval.

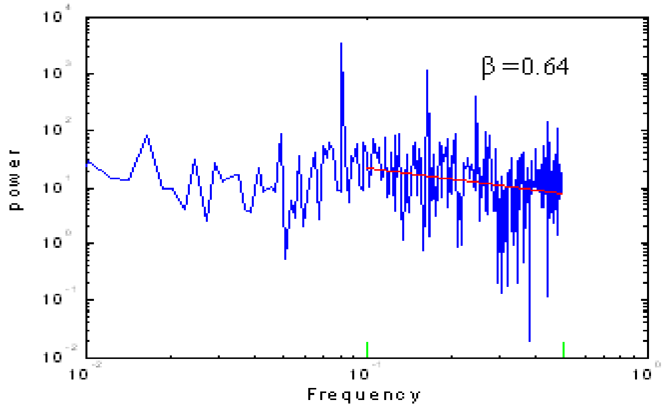

Fig. 4: Power spectrum for monthly rainfall in the Ramsar.

Time lag and Embedding dimension:

In this study τ is computed using the AMI method using time lags of 1-100 month. The AMI shows well-defined first minima at time lag 4 month (Fig.5). Hence, the optimal embedding delay τ_{opt} is chosen as 13 for our analysis. The method used for the determination of the sufficient embedding dimension is based on the calculation of the percentage of false nearest-neighbours for the time series. In Fig.6 we show the density of FNN *vs*. the embedding dimension *m* for monthly time series: the optimal embedding is chosen to be *m* = 13.

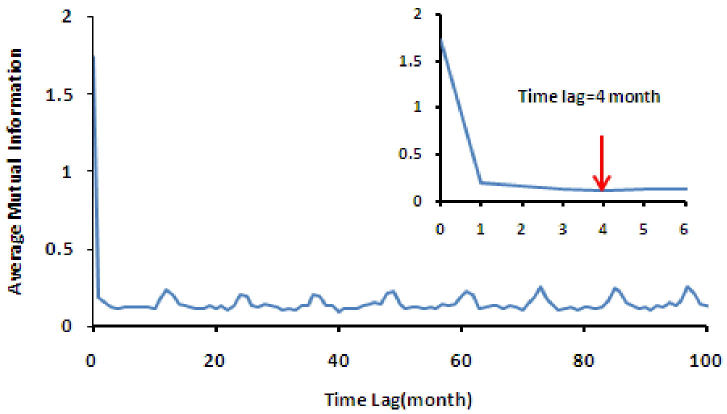

Fig. 5: Average mutual information (AMI) function of the rainfall time series.

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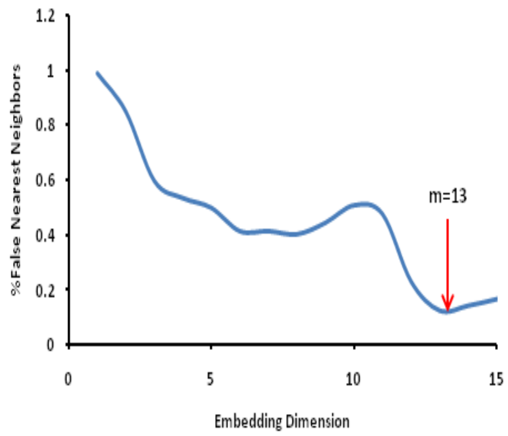

Fig. 6: Percentage of false nearest neighbour (FNN) of rainfall time series in embedding dimension.

Correlation dimension:

The correlation function calculated for the dataset using the delay times (τ =4), determined by the AMI method in the previous section, and for embedding dimensions, *m*, from 1 to 20. Figure 7 shows the relationship between the correlation function *C(r)* and the radius *r* (i.e. *lnC(r)* versus *lnr*) for increasing *m.*

The relationship between the correlation dimension values $D_2(m)$ and the embedding dimension values *m* is shown in Fig.8.

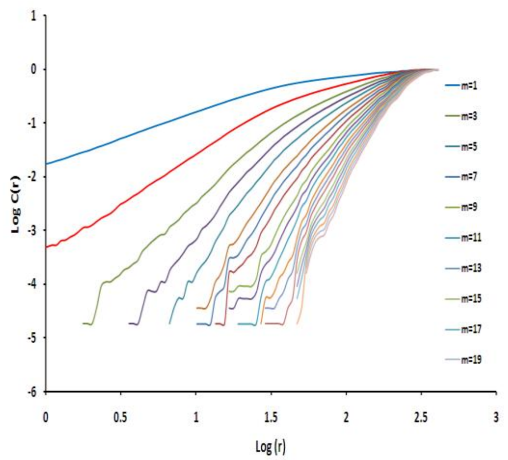

Fig. 7: *lnC(r)* versus *ln r* plots for rainfall data from the Ramsar.

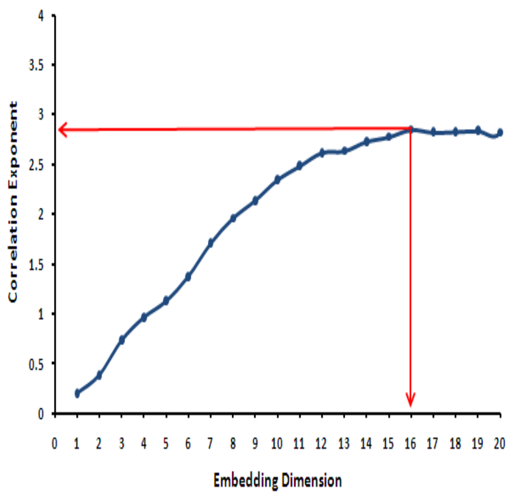

Fig. 8: Relation between correlation exponent *D2* and embedding dimension *m.*

It can be seen that the value of correlation exponent increases with the embedding dimension up to a certain value and then saturates beyond it. The saturation of the correlation exponent beyond a certain embedding dimension value is the indication of an existence of deterministic dynamics. The saturated correlation dimension is ~ 2.8, ($D_2 = 2.82$). The value of correlation dimension suggests the possible presence of chaotic behaviour and fractal characteristics in the rainfall time series. Because correlation dimension for the monthly rainfall is above 2, at least three independent variables are needed to describe the dynamics of rainfall of the Ramsar. This also is taken as the minimum dimension of the phase space that can embed the attractor.

Largest Lyapunov exponent:

We apply the method of estimation of the largest Lyapunov exponent described above to rainfall time series for the Ramsar data, using the same delay time and embedding dimension as before. Fig.9 shows the curve for the stretching factor *S* versus the number of points *N*. The slope value corresponding to the largest Lyapunov exponent is obtained after the least-squares line fit for the Rainfall time series and is found to be 0.0135. This positive value indicates a strong signature of chaos. Inverse the largest Lyapunov exponent estimates predictability of time series, so it shows the monthly Rainfall in Ramsar is predictably for 74 next months.

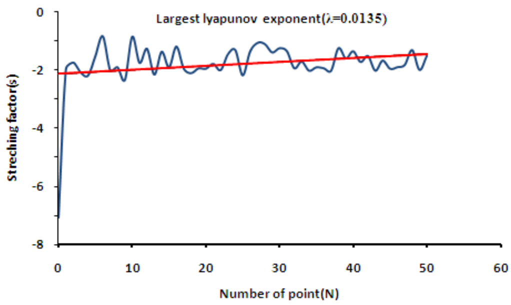

Fig. 9: Estimation of the largest Lyapunov exponent using the method of Rosenstein *et al*. (1993).

Conclusion:

This study investigated the existence of chaotic signals in the monthly rainfall in Ramsar, Iran. The power spectrum, average mutual information approach, the false nearest neighbor algorithm, the correlation integral analysis and the Lyapunov exponent analysis were used in the research by TISEAN package (Hegger *et al*.,1999). The results show that the value of the power spectrum indicates that chaotic (fractal) behavior to rainfall time series. The mutual information approach and the false nearest neighbor algorithm provided a time lag and embedding dimension which is needed to reconstruct phase space. The correlation dimension method provided a low fractal-dimensional attractor thus suggesting a possibility of the existence of chaotic signals. Based on the attractor dimensions, the minimum number of variables essential to model the monthly rainfall dynamics was identified as 3.Finally the positive largest Lyapunov exponent indicates a existence of chaos behavior and it is predictably for 74 next months.

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