



AENSI Journals

Journal of Applied Science and Agriculture

ISSN 1816-9112

Journal home page: www.aensiweb.com/jasa/index.html



A New Fuzzy TOPSIS Ranking Method Based On Point To Point Interval Type-2 Fuzzy Sets (PTP IT2 FS) In MCDM Problems

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ARTICLE INFO

Article history:

Received 20 January, 2014

Received in Revised form 16 April, 2014

Accepted 25 April 2014

Available Online 5 May, 2014

Key words:

MCDM, Fuzzy Ranking, Point to point interval type-2 fuzzy sets (PTP IT2 FS).

ABSTRACT

Decisions in the public and individually sector decision making often involve the assessment and ranking of decision alternatives based on multi criteria decision making (MCDM). Fuzzy Technique for Order Preference by Similarly to Ideal Solution (TOPSIS) is one of the most commonly used approaches in solving numerous multi criteria decision making problems. It has been widely used in ranking of multi alternatives with respect to multi criteria with the superiority of fuzzy set type-1. In this paper based on the interval type-2 fuzzy sets, we introduce a new method of fuzzy TOPSIS for handling fuzzy ranking multi criteria group decision making problems. In the proposed method we obtain type-1 fuzzy set for each one of the membership function in the part of linguistic variable for the rating of alternatives and the importance weight of each criterion in the form of point to point interval type-2 fuzzy sets (PTP IT2 FS). To illustrate the performance of the proposed method, the weights and the rate of alternatives decision making problem is solved in section 3. At the end of paper results demonstrate that this method provides us with a useful way to handle the fuzzy multi criteria group decision making problems in a more flexible and more intelligent manner due to the fact that it uses PTP IT2 FS rather than traditional type-1 fuzzy sets.

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To Cite This Article: Shahin Saeednamaghi and Assef Zare., A New Fuzzy TOPSIS Ranking Method Based On Point To Point Interval Type-2 Fuzzy Sets (PTP IT2 FS) In MCDM Problems. *J. Appl. Sci. & Agric.*, 9(4): 1379-1388, 2014

INTRODUCTION

One of the well known methods in multi criteria decision making (MCDM) is Technique for Order Preference by Similarly to Ideal Solution (TOPSIS). The technique was developed by Hwang and Yoon (Hwang, C.L. and K. Yoon, 1981). TOPSIS has become a favorable technique in solving MCDM problems because the concept is reasonable, easy to understand and it needs less computational effort (Abdullah, L., A. Otheman). The basic concept of this method is the selected alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. In TOPSIS the performance ratings and the weights of the attributes are given as crisp values (Ashtiani, B., F. Haghighirad, 2009). There are a bunch of research articles on fuzzy TOPSIS for MCDM, see references (Saeednamaghi, S.H., A. Zare, 2014. Chen, C.T., 2000). Introduced an extension of TOPSIS under fuzzy environment. Triantaphyllou and Lin (Triantaphyllou, E., C.T. Lin, 1996) developed a fuzzy version of TOPSIS method based on fuzzy arithmetic operations. Wang and Lee (Wang, Y.J., H.S. Lee, 2007), generalized TOPSIS to fuzzy multi criteria group decision making (FMCGDM) in a fuzzy environment. Ashtiani et al., (2009). Introduced an extension of fuzzy TOPSIS based on interval-valued fuzzy sets. Chen and Lee (Chen, S.M., L.W. Lee, 2010), developed TOPSIS for fuzzy multiple attributes group decision making based on the interval type-2 fuzzy sets. Exact data may be difficult to be precisely determined since human judgments are often vague under many conditions (Chen, T.Y., 2011. Nurnadiah, Z., A. Lazim, 2012). Thus, an extension of TOPSIS to the fuzzy environment is a natural generalization of TOPSIS methods (Jahanshahloo, G.R. et al, 2006; Baguley, P., T. Page, 2006; Nurnadiah, Z., A. Lazim, 2012). However, fuzzy TOPSIS is still believed not suitable to represent the uncertainties. Zade (Zade, L.A., 1975) was developed a new fuzzy set method known as type-2 fuzzy set. Then Mendel et al. (2006) Nurnadiah, Z., A. Lazim, 2012) improved this type-2 fuzzy set into an interval type-2 fuzzy set. In (Wu, D., J.M. Mendel, 2007), Wu and Mendel pointed out that interval type-2 fuzzy sets have been applied in many areas, such as: decision making (Mendel J.M., 2002; Ozen, T., J.M. Garibaldi, 2004; Sevastjanov, P., P. Figat, 2007; Wu, D., J.M. Mendel, 2007; Wu, H., J.M. Mendel, 2004), time series forecasting (Baguley, P., T. Page, 2006, Karnik, N.N., J.M. Mendel; Mendel, J.M., 2001), survey processing (Auephanwiriyakul, S., A. Adrian; 2002.

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Liu, F., J.M. Mendel, 2007, Mendel, J.M., 2001), document retrieval (Castillo, O., N. Cazarez), speech recognition (Melin, P., J. Urias, 2006; Zeng, J., Z.O. Liu, 2006), noise cancellation (Castillo, O., P. Melin, 2004; Castillo, O., P. Melin, 2008, Own, C.M., H.H.Tsai, 2006), word modeling (Liu, F., J.M. Mendel, 2007; Mendel, J.M., H. Wu, 2004; Wu, H., J.M. Mendel, 2004), clustering (Rhee, F.C.H., 2007), control (Astudillo, L., O. Castillo, 2007; Castillo, O., N. Cazarez, 2006; Castillo, O., P. Melin, 2005; Castillo, O., P. Melin, 2008; Figueroa, J., J. Posada, 2005; Hagraas, H., 2007; Lee, C.H., Y.C. Lin, 2004; Lin, P.Z., C.F. Hsu, 2005; Lynch, C., H. Hagraas, 2006; Melin, P., O. Castillo, 2007; Sepulveda, R., O. Castillo, 2007; Wu, D., W.W. Tan, 2006; Wu, D., W.W. Tan, 2006), wireless communication (Liang, Q., L. Wang, 2005; Shu, H., Q. Liang, 2005) web-shopping (Gu L., Y.Q. Zhang, 2007) and linguistic summarization of databases (Niewiadomski, A., M. Bartyzel, 2006; Niewiadomski, A., P.S. Szczepaniak, 2006). Although T2FS have been widely applied in many areas, most researchers used interval T2FS (IT2FS) in practical fields because of the computational complexity involved in using T2FS (Zhang, Z., 2012) Zhang introduced interval type-2 rough fuzzy set as a combination of IT2FS and rough set theory. He also manage to get the relationship between IT2FS and interval type 2 fuzzy topology operators. Until now, there are a lot of studies discussed on the improvement of ranking type-2 fuzzy numbers. For example, Dongrui Wu, Jerry M. Mendel (2009) proposed a new ranking method and a new similarity measure for IT2 FSs. Chen Shyi-Ming, Wang Cheng-Yi (2013) present a new method for fuzzy multiple attributes decision making based on interval type-2 fuzzy sets. Their proposed method can overcome the drawbacks of Liu and Su's method (2012) and Wang and Luo's method (Wang, Y.M., Y. Luo, 2009) due to the fact that it can deal with the ranking of interval type-2 fuzzy sets to distinguish the preference order of the alternatives. Chen and Lee (2010) presented a method for fuzzy multiple attributes group decision making based on the ranking values and the arithmetic operations of interval type-2 fuzzy sets. Chen and Lee (2010) presented a method for fuzzy multiple attributes group decision making based on the interval type-2 TOPSIS method. (Huynh et al.2008) presented a method for ranking fuzzy numbers based on the probability-based comparison relation of fuzzy numbers and then applied the fuzzy ranking method for target-based fuzzy decision making. (Yager, R.R., 1999) presented a method for fuzzy multiple attributes decision making based on fuzzy probability assessments. Yager (Yager, R.R., 1999) presented a method for fuzzy multiple attributes decision making by including the decision-maker's attitude based on comparing fuzzy sets. (Yager, R.R., 2000) presented a method for fuzzy multiple attributes decision making based on evaluating fuzzy sets and the attitude of the decision-maker. The rest of this paper is organized as follows: In Section 2, we briefly review some basic concepts of interval type-2 fuzzy sets. In Section 3, we propose a new fuzzy ranking method based on point to point interval type-2 fuzzy sets (PTP IT2 FS) with considering of decision-maker's alternatives and weights. Then, based on the proposed method, we design type-1 fuzzy sets an example, see (Saeednamaghi, S.H., A. Zare, 2014). This method provided decision makers (DMs) more information to make more suitable decision and easy to understand and it needs less computational effort. In Section 4, we compare our results method with type-1 fuzzy an example mentioned in section 3 .Finally, the conclusions are discussed in Section 5.

2. Interval Type-2 Fuzzy Sets: Definitions:

For convenience in defining the Upper and Lower MFs (Membership Functions) of a type-2 MF, we first give the definition of footprint of uncertainty (FOU) of a type-2 MF.

Definition 2.1 (Footprint of Uncertainty of a Type-2 MF):

Uncertainty in the primary membership grades of a type-2 MF consists of a bounded region that we call the footprint of uncertainty of a type-2 MF (e.g., see Fig. 1). It is the union of all primary membership grades.

Definition 2.2 (Upper and Lower MFs):

An Upper MF (UMF) and a Lower MF (LMF) are two type-1 MFs that are bounds for the footprint of uncertainty of an interval type-2 MF. The Upper MF is a subset that has the maximum membership grade of the footprint of uncertainty; and the Lower MF is a subset that has the minimum membership grade of the footprint of uncertainty.

3. The propose a new fuzzy ranking method based on point to point interval type-2 fuzzy sets:

In this Section, we propose a new fuzzy ranking method based on point to point interval type-2 fuzzy sets with the consideration of the decision-maker s alternatives and weights. Saeednamaghi SH, and Zare A. (2014) proposed fuzzy TOPSIS algorithm for multi criteria decision making with application of Marketing Mix under α -cut. Now we present a new fuzzy ranking method based on interval type-2 fuzzy sets for their alternatives and weights that they used type-1 fuzzy sets in their paper. They used Table I and Table II for Linguistic variables for the rating of alternatives and Linguistic variables for importance weight of each criterion.

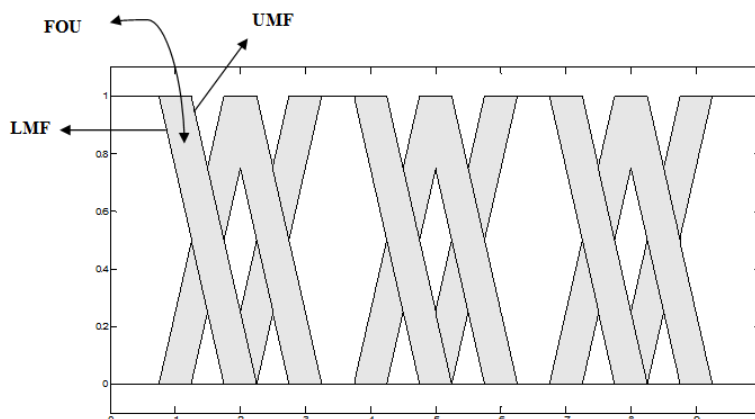


Fig. 1: Interval type-2 fuzzy set (IT2 FS) and associated quantities for triangular and trapezoidal MFs.

Table I: Linguistic variables for the rating of alternatives.

Linguistic variable	fuzzy number
Very Poor (VP)	(0,1,1,67,3)
Poor (P)	(1,5,5,7,8)
Medium Poor (MP)	(2,7,67,8,9)
Fair (F)	(5,7,7,67,9)
Medium Good (MG)	(4,7,67,8,9)
Good (G)	(3,8,33,9,33,10)
Very Good (VG)	(6,8,67,10,10)

Table II: Linguistic variables for importance weight of each criterion.

Linguistic variable	Corresponding trapezoid fuzzy number
Very Low (VL)	(0,0,1,2)
Low (L)	(1,2,2,3)
Medium Low (ML)	(2,3,4,5)
Medium (M)	(4,5,5,6)
Medium High (MH)	(5,6,7,8)
High (H)	(7,8,8,9)
Very High (VH)	(8,9,10,10)

In the point to point interval type-2 fuzzy problems, first suppose that we have a type-1 fuzzy set in the following form:

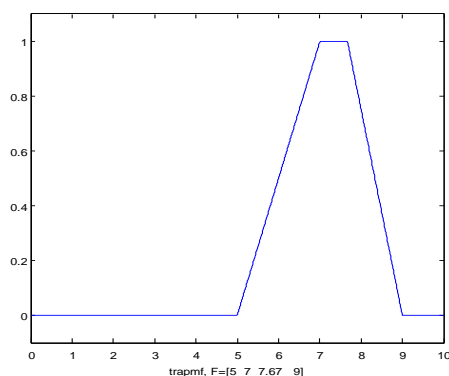


Fig. 2: A type-1 fuzzy set.

Now for each one of the fourfold points of membership function of the mentioned pattern, we consider the distribution amount in the way we have:

In the numerous existed environment (Footprint Of Uncertainty (FOU)) the random type-1 membership function it can be imagined. For example that:

In this way first for each one of the membership function in the part of linguistic variable for the rating of alternatives and the importance weight of each criterion, the distribution amount is considered and then by the atmosphere made (FOU) for each linguistic variable, we choose randomly the membership function and solve the problem based on produced linguistic variables. If we repeat the mentioned process for N- times and finally mean it, we can obtain a good average by solving based on type-2 fuzzy system.

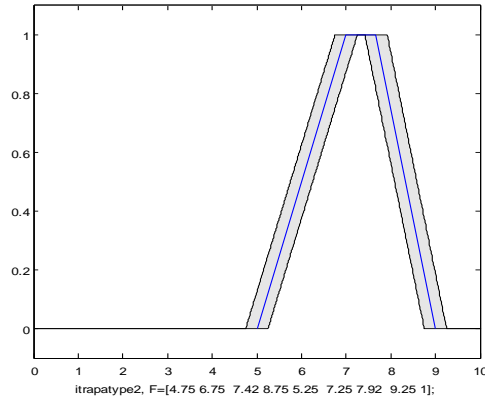


Fig. 3: Membership function of the mentioned pattern.

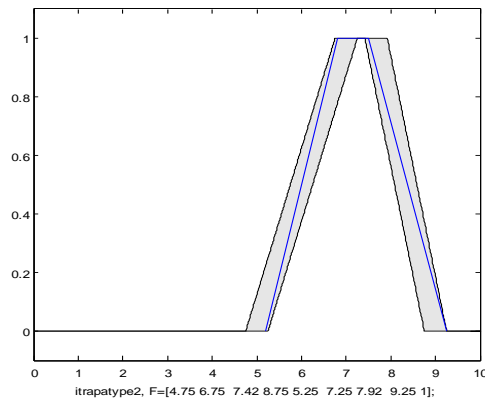


Fig. 4: The random type-1 membership function in FOU.

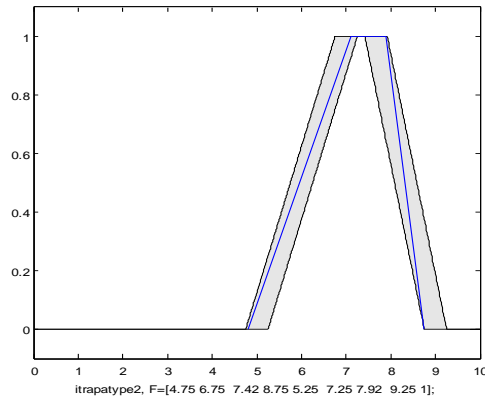


Fig. 5: The other random type-1 membership function in FOU.

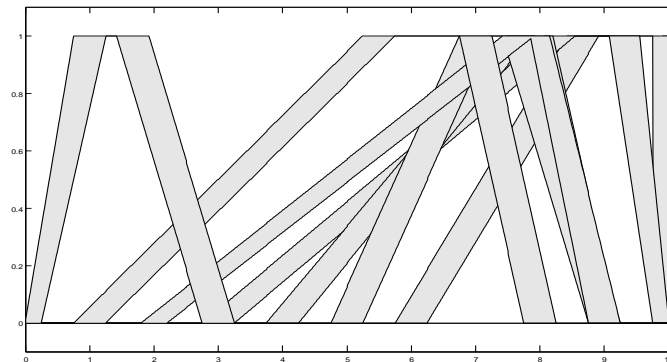


Fig. 6: The linguistic variable for the rating of alternatives of each criterion.

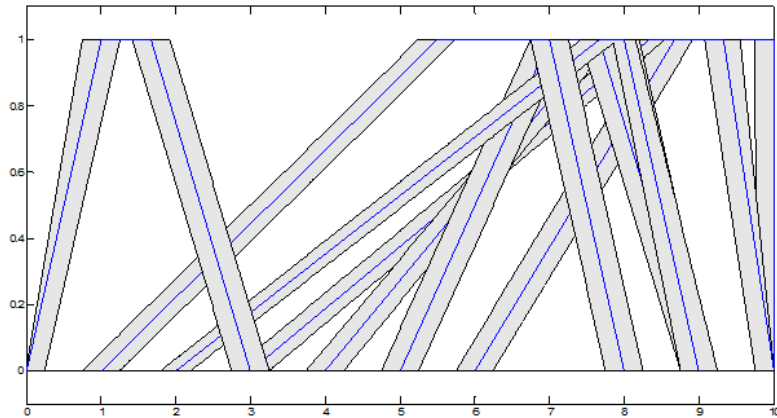


Fig. 7: The linguistic variable for the rating of alternatives of each criterion with the choice of type-1 membership function.

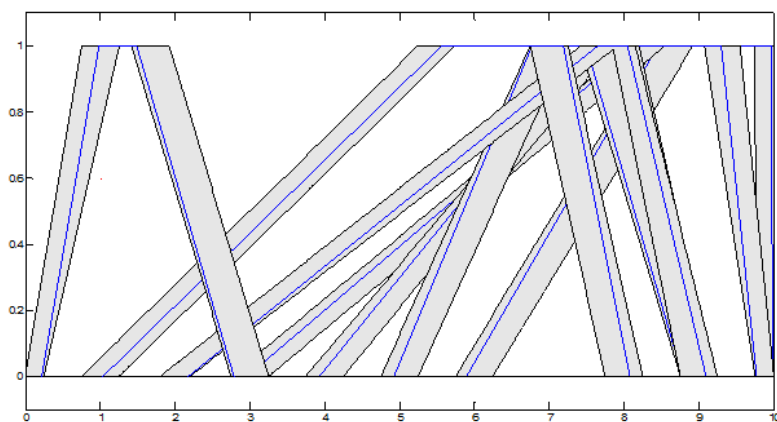


Fig. 8: The linguistic variable for the rating of alternatives of each criterion with the choice of type-1 membership function in random form have.

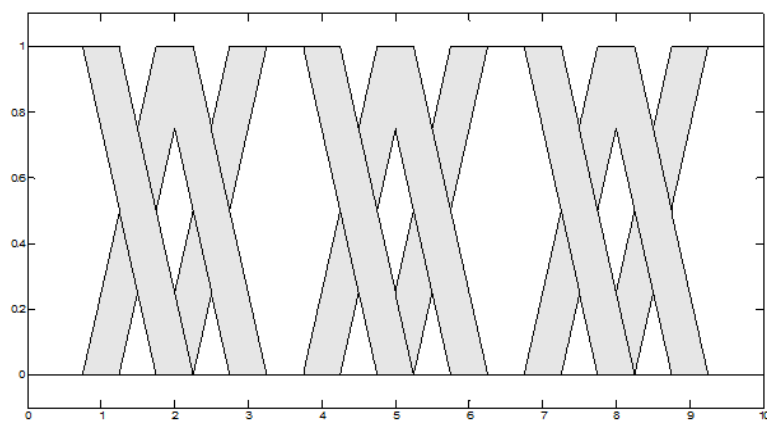


Fig. 9: The linguistic variables for the importance weight of each criterion.

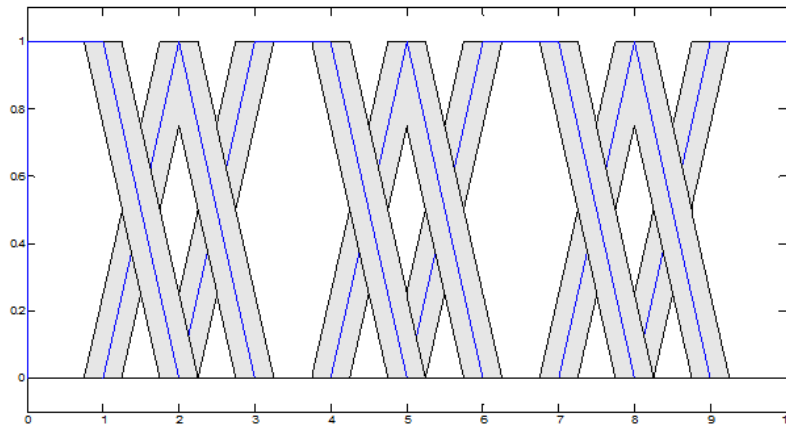


Fig. 10: The linguistic variables for the importance weight of each criterion with the choice of type-1 membership function .

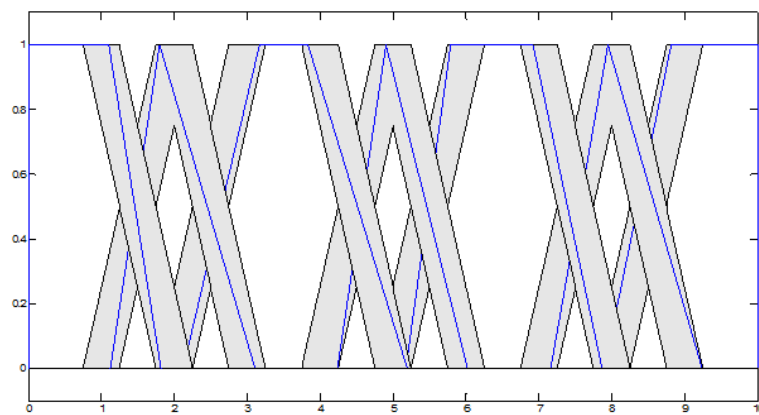


Fig. 11: The linguistic variables for the importance weight of each criterion with the choice of type-1 membership function in random form have.

Finally, been compared the comparison result (Closeness Coefficient (CC)) between random type-1 fuzzy TOPSIS ranking in FOU and point to point interval type-2 fuzzy TOPSIS ranking (for meaning 100 times solving in the case of type-1 fuzzy), that the obtained result will be compared with type-1 fuzzy TOPSIS ranking in (Saeednamaghi, S.H., A. Zare, 2014) in the following Table III and Fig. 12. The only difference is observed in ranking of fifth and sixth alternatives.

For meaning 100 times solving in the case of type-1 fuzzy, we use this under method for all alternatives each time as below :

The $CC_{PTPIT2FS}$ and $CC_{random T1 fuzzy in FOU}$ is calculated as below :

$$CC_K = \frac{1}{\sum_{\alpha} [CC_i|_{\alpha=0} * (0) + CC_i|_{\alpha=0.25} * (0.25) + CC_i|_{\alpha=0.5} * (0.5) + CC_i|_{\alpha=0.75} * (0.75) + CC_i|_{\alpha=1} * (1)]}$$

$$\alpha = (0, 0.25, 0.5, 0.75, 1), (i = 1, \dots, m), (m = 15), (K = 1, \dots, 100)$$

$$CC_{PTPIT2FS} = \frac{\sum_{K=1}^{100} CC_K}{100}, \quad CC_{random T1 fuzzy in FOU} = CC_K \quad (K = 1)$$

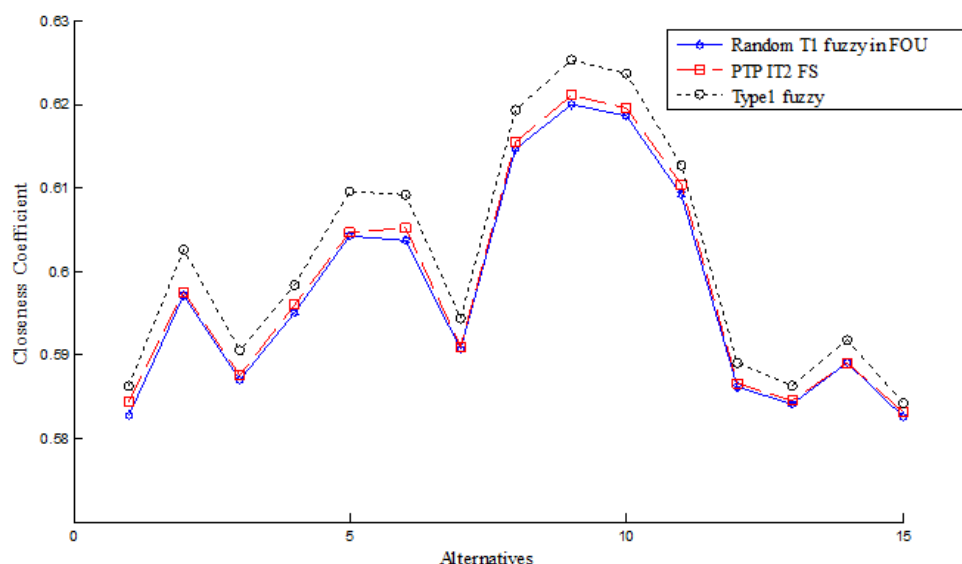
The $CC_{PTPIT2FS}$ and $CC_{random T1 fuzzy in FOU}$ will be obtained for the first Alternative as below :

$$CC_{\text{random T1 fuzzy in FOU}} = 0.5828 \quad \& \quad CC_{\text{PTPIT2FS}} = 0.5844$$

Table III: Comparison results.

Alternatives	CC random T1 fuzzy in FOU	Ranking random T1 fuzzy in FOU	CC PTP IT2 FS	Ranking PTP IT2 FS	CC T1 fuzzy Saeednamaghi (2014)	Ranking T1 fuzzy Saeednamaghi (2014)
1	0.5828	14	0.5844	14	0.5863	14
2	0.5971	7	0.5975	7	0.6026	7
3	0.5870	11	0.5875	11	0.5906	11
4	0.5951	8	0.5961	8	0.5983	8
5	0.6042	5	0.6046	6	0.6095	5
6	0.6037	6	0.6052	5	0.6092	6
7	0.5907	9	0.5910	9	0.5944	9
8	0.6146	3	0.6154	3	0.6193	3
9	0.6199	1	0.6211	1	0.6253	1
10	0.6186	2	0.6195	2	0.6237	2
11	0.6091	4	0.6104	4	0.6127	4
12	0.5862	12	0.5866	12	0.5891	12
13	0.5841	13	0.5846	13	0.5864	13
14	0.5890	10	0.5891	10	0.5918	10
15	0.5827	15	0.5832	15	0.5843	15

(CC=Closeness Coefficient)

**Fig. 12:** Comparison results.**Conclusion:**

In this paper, we have presented a new method for ranking fuzzy multi criteria decision making based on point to point interval type-2 fuzzy sets (PTP IT2 FS). This method, which can easily be re-done for new data sets that a reader collects, should help people better understand the uncertainties associated with linguistic terms and hence how to use the uncertainties effectively in survey design and linguistic information processing. The proposed method is more flexible and more intelligent in the ranking of alternatives. The example given in this paper illustrates that the proposed method has the distinct characteristics. Additionally, the proposed method can provide assurance for decision makers by ranking alternatives in different fields that is, for many such problems, the decision maker wants to solve a multi criteria decision making (MCDM) problem such as: Marketing Mix, management, business and so on. Finally, we compared the Closeness Coefficient between type-1 fuzzy TOPSIS ranking method mentioned in (Saeednamaghi, S.H., A. Zare, 2014) with point to point interval type-2 fuzzy TOPSIS ranking and random type-1 fuzzy TOPSIS in footprint of uncertainty (FOU) for all alternatives.

REFERENCES

- Abdullah, L., A. Otheman, 2013. I.J. Intelligent Systems and Applications, 02: 25-33.
- Ashtiani, B., F. Haghighirad, G.A. Montazer, 2009. Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets, Applied Soft Computing, 9: 457-461.
- Astudillo, L., O. Castillo, L.T. Aguilar, 2007. Intelligent control for a perturbed autonomous wheeled mobile robot: a type-2 fuzzy logic approach, Journal of Nonlinear Studies, 14(3): 37-48.
- Auephanwiriyakul, S., A. Adrian, J.M. Keller, 2002. Type-2 fuzzy set analysis in management surveys, in: Proceedings of the 2002 IEEE International Conference on Fuzzy Systems, Honolulu, HI, pp: 1321-1325.
- Baguley, P., T. Page, V. Koliza, P. Maropoulos, 2006. Time to market prediction using type-2 fuzzy sets, Journal of Manufacturing Technology Management 17(4): 513-520.
- Berensten et al., 2008. On cheating doping and whistle blowing. Eur. J. Political Eco, 24: 415-436.
- Bouchachia, A., R. Mittermeir, 2003. A neural cascade architecture for document retrieval, in: Proceedings of the International Joint Conference Neural Networks, Portland, OR, 3:1915-1920.
- Castillo, O., N. Cazarez, P. Melin, 2006. Design of stable type-2 fuzzy logic controllers based on a fuzzy Lyapunov approach, in: Proceedings of the 2006 IEEE International Conference on Fuzzy Systems, Vancouver, Canada, pp: 2331-2336.
- Castillo, O., P. Melin, 2004. Adaptive noise cancellation using type-2 fuzzy logic and neural networks, in: Proceedings of the 2004 IEEE International Conference on Fuzzy Systems, 2: 1093-1098.
- Castillo, O., P. Melin, 2005. Evolutionary computing for optimizing type-2 fuzzy logic systems in intelligent control of non-linear dynamic plants, in: Proceedings of the North American Fuzzy Information Processing Society (NAFIPS), Ann Arbor, MI, pp: 247-251.
- Castillo, O., P. Melin, 2008. In Kacprzyk J. (Ed.), Type-2 Fuzzy Logic: Theory and Applications, Springer-Verlag, Berlin, Heidelberg.
- Chen, C.T., 2000. Extensions of the TOPSIS for group decision-making under fuzzy environment, Fuzzy Sets and Systems, 11: 41-9.
- Chen, Shyi-Ming, 2013. Wang Cheng-Yi. Fuzzy decision making systems based on interval type-2 fuzzy sets. Information Sciences, 242: 1-21.
- Chen, S.M., L.W. Lee, 2010. Fuzzy multiple attributes group decision-making based on the interval type-2 TOPSIS method, Expert systems with applications, 37: 2790-2798.
- Chen, S.M., L.W. Lee, 2010. Fuzzy multiple attributes group decision-making based on the ranking values and the arithmetic operations of interval type-2 fuzzy sets, Expert Systems with Applications, 37(1): 824-833.
- Chen, T.Y., 2011. Interval-valued fuzzy TOPSIS method with leniency reduction and an experimental analysis. J. Applied Soft Compute, 11:4591-4606.
- Dongrui Wu, J.M. Mendel, 2009. A comparative study of ranking methods, similarity measures and uncertainty measures for interval type-2 fuzzy sets. Information Sciences, 179: 1169-1192.
- Figuroa, J., J. Posada, J. Soriano, M. Melgarejo, S. Rojas, 2005. A type-2 fuzzy controller for tracking mobile objects in the context of robotic soccer games, in: Proceedings of the 2005 IEEE International Conference on Fuzzy Systems, Reno, NV, pp: 359-364.
- Gu L., Y.Q. Zhang, 2007. Web shopping expert using new interval type-2 fuzzy reasoning, Soft Computing, 11(8): 741-751.
- Hagras, H., 2007. Type-2 FLCs: a new generation of fuzzy controllers, IEEE Computational Intelligence Magazine, 2(1): 30-43.
- Huynh, V.N., Y. Nakamori, J. Lawry, 2008. A probability-based approach to comparison of fuzzy numbers and applications to target-oriented decision making, IEEE Transactions on Fuzzy Systems, 16(2): 371-387.
- Hwang, C.L. and K. Yoon, 1981. Multiple Attributes Decision Making Methods and Applications. Springer-Verlag, Berlin.
- Jahanshahloo, G.R. et al, 2006. Extension of the TOPSIS method for decision-making problems with fuzzy data. Appl Math Comput, 181: 1544-1551.
- Karnik, N.N., J.M. Mendel, Q. Liang, 1999. Type-2 fuzzy logic systems, IEEE Transactions on Fuzzy Systems, 7(6): 643-658.
- Abdullah, L., A. Otheman, A New Entropy Weight for Sub-Criteria in Interval Type-2 Fuzzy TOPSIS and Its Application, I.J., 2013. Intelligent Systems and Applications, 02: 25-33.
- Lee, C.H., Y.C. Lin, 2004. Control of nonlinear uncertain systems using type-2 fuzzy neural network and adaptive filter, in: Proceedings of the 2004 IEEE International Conference on Networking, Sensing and Control, Taipei, Taiwan, 2: 1177-1182.
- Liang, Q., L. Wang, 2005. Sensed signal strength forecasting for wireless sensors using interval type-2 fuzzy logic systems, in: Proceedings of the 2005 IEEE International Conference on Fuzzy Systems, Reno, NV, pp: 25-30.

- Liu, P., Y. Su, 2012. Multiple attribute decision making method based on the trapezoid fuzzy linguistic hybrid harmonic averaging operator, *Informatica*, 36(1): 83-90.
- Lin, P.Z., C.F. Hsu, T.T. Lee, 2005. Type-2 fuzzy logic controller design for buck DC-DC converters, in: *Proceedings of the 2005 IEEE International Conference on Fuzzy Systems*, Reno, NV, pp: 365-370.
- Liu, F., J.M. Mendel, 2007. An interval approach to fuzzistics for interval type-2 fuzzy sets, in: *Proceedings of the 2007 IEEE International Conference on Fuzzy Systems*, UK, pp: 1-6.
- Liu, Xinwang. et al., 2012. Analytical solution methods for the fuzzy weighted average. *Information Sciences*, 187: 151-170.
- Lynch, C., H. Hagrass, V. Callaghan, 2006. Using uncertainty bounds in the design of an embedded real-time type-2 neuro-fuzzy speed controller for marine diesel engines, in: *Proceedings of the 2006 IEEE International Conference on Fuzzy Systems*, Vancouver, Canada, pp: 7217-7224.
- Melin, P., O. Castillo, 2007. An intelligent hybrid approach for industrial quality control combining neural networks, type-2 fuzzy logic and fractal theory, *Information Sciences*, 177(7): 1543-1557.
- Melin, P., J. Urias, D. Solano, M. Soto, M. Lopez, O. Castillo, 2006. Voice recognition with neural networks, type-2 fuzzy logic and genetic algorithms, *Journal of Engineering Letters*, 13(2): 108-116.
- Mendel J.M., 2002. An architecture for making judgments using computing with words, *International Journal of Applied Mathematics and Computer Science*, 12(3): 325-335.
- Mendel, J.M., 2001. *Uncertain Rule-Based Fuzzy Logic Systems: Introduction and New Directions*, Prentice-Hall, Upper Saddle River, NJ.
- Mendel, J.M., H. Wu, 2004. Centroid uncertainty bounds for interval type-2 fuzzy sets: forward and inverse problems, in: *Proceedings of the 2004 IEEE International Conference on Fuzzy Systems*, vol. 2, Budapest, Hungary, pp: 947-952.
- Mendle, et al., 2006. Interval type-2 fuzzy logical systems made simple. *IEEE Trans. Fuzzy Syst.*, 14: 808-821.
- Niewiadomski, A., M. Bartyzel, 2006. Elements of type-2 semantics in summarizing databases, in: *Proceedings of the 8th International Conference on Artificial Intelligence and Soft Computing*, 4029: 278-287.
- Niewiadomski, A., P.S. Szczepaniak, 2006. News generating based on type-2 linguistic summaries of databases, in: *Proceedings of the 2006 International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems*, Paris, France, pp: 1324-1331.
- Nurnadiyah, Z., A. Lazim, 2012. Weight of interval type-2 fuzzy Rasch Model in decision making approach: ranking causes lead of road accident occurrence. *International Journal of Soft Computing*, 7(1): 1-11.
- Own, C.M., H.H. Tsai, P.T. Yu, Y.J. Lee, 2006. Adaptive type-2 fuzzy median filter design for removal of impulse noise, *Imaging Science*, 54(1): 3-18.
- Ozen, T., J.M. Garibaldi, 2004. Effect of type-2 fuzzy membership function shape on modeling variation in human decision making, in: *Proceedings of the 2004 IEEE International Conference on Fuzzy Systems*, Budapest, Hungary, pp: 971-976.
- Rhee, F.C.H., 2007. Uncertainty fuzzy clustering: insights and recommendations, *IEEE Computational Intelligence Magazine*, 2(1): 44-56.
- Saeednamaghi, S.H., A. Zare, 2014. *Global Journal of Engineering, Design & Technology*. G. J. E.D.T. 3(1): 29-37.
- Sepulveda, R., O. Castillo, P. Melin, A. Rodriguez-Diaz, O. Montiel, 2007. Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic, *Information Sciences*, 177(10): 2023-2048.
- Sevastjanov, P., P. Figat, 2007. Aggregation of aggregating modes in MCDM: synthesis of type 2 and level 2 fuzzy sets, *Omega*, 35(5): 505-523.
- Shu, H., Q. Liang, 2005. Wireless sensor network lifetime analysis using interval type-2 fuzzy logic systems, in: *Proceedings of the 2005 IEEE International Conference on Fuzzy Systems*, Reno, NV, pp: 19-24.
- Triantphyllou, E., C.T. Lin, 1996. Development and evaluation of five fuzzy multi attribute decision making methods, *International Journal of Approximate Reasoning*, 14: 281-310.
- Wang, Y.J., H.S. Lee, 2007. Generalizing TOPSIS for fuzzy multiple-criteria group decision making, *An international computing and mathematics with applications*, 53: 1762-1772.
- Wang, Y.M., Y. Luo, 2009. Area ranking of fuzzy numbers based on positive and negative ideal points, *Computers and Mathematics with Applications*, 58(9): 1769-1779.
- Wu, D., J.M. Mendel, 2007. Aggregation using the linguistic weighted average and interval type-2 fuzzy sets, *IEEE Transactions on Fuzzy Systems*, 15(6): 1145-1666.
- Wu, D., J.M. Mendel, 2007. Uncertainty measures for interval type-2 fuzzy sets, *Information Sciences*, 177(23): 5378-5393.
- Wu, D., W.W. Tan, 2006. A simplified type-2 fuzzy controller for real-time control, *ISA Transactions*, 45(4): 503-516.
- Wu, D., W.W. Tan, 2006. Genetic learning and performance evaluation of type-2 fuzzy logic controllers, *Engineering Applications of Artificial Intelligence*, 19(8): 829-841.

Wu, H., J.M. Mendel, 2004. Antecedent connector word models for interval type-2 fuzzy logic systems, in: Proceedings of the 2004 IEEE International Conference on Fuzzy Systems, Budapest, Hungary, 2: 1099-1104.

Yager, R.R., 1999. Decision making with fuzzy probability assessments, IEEE Transactions on Fuzzy Systems, 7(4): 462-467.

Yager, R.R., 2000. Fuzzy modeling for intelligent decision making under uncertainty, IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 30 (1): 60-70.

Yager, R.R., 1980. Fuzzy subsets of type-2 in decisions, Journal of Cybernetics, 10: 137-159.

Yager, R.R., 1999. Including decision attitude in probabilistic decision making, International Journal of Approximate Reasoning, 21(1): 1-21.

Zade, L.A., 1975. The concept of linguistic variable and its application to approximate reasoning-I. Inform. Sci., 8: 199-249.

Zeng, J., Z.O. Liu, 2006. Type-2 fuzzy hidden Markov models and their applications to speech recognition, IEEE Transactions on Fuzzy Systems, 14(3): 454-467.

Zhang, Z., 2012. On Interval Type-2 Rough Fuzzy Sets. Knowl. Based System. <http://dx.doi.org/10.1016/j.knosys>.