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Comparison in Limit Equilibrium Methods of Slices in Slope Stability Analysis

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ABSTRACT

The geotechnical engineer frequently uses limit equilibrium methods of analysis when studying slope stability problems. The paper compares five methods of slices commonly used for slope stability analysis, because of its simplicity and accuracy. These methods are Ordinary, Simplified Janbu, Simplified Bishop, Spencer and Morgenstern-Price's method. The factor of safety equations are written in the same form, recognizing whether moment and (or) force equilibrium is explicitly satisfied. At last, a complete comparison between the results of these methods will be done.

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INTRODUCTION

There are various methods for slope stability analysis, the geotechnical engineer frequently uses limit equilibrium methods for analysis when studying slope stability problems, because of its simplicity and accuracy. To slope stability analysis by limit equilibrium method there are two considerations. (1) The method of wedge and (2) the method of slices. The methods of slices have become the most common methods due to their ability to accommodate complex geometrics and variable soil and water pressure conditions. During the past three decades approximately one dozen methods of slices have been developed (Fellenius, 1936:445-462; Zhu DY, 2005: 272–278). They differ in these methods in (i) the statics employed in deriving the factor of safety equation and (ii) the assumption used to render the problem determinate.

One of the important researches in the past few decades is Fredlund and Krahn(1977) (Fredlund, 1977:429-439.). In this paper, a compare in the various methods of slices in terms of consistent procedures for deriving the factor of safety equations. All equations are extended to the case of a composite failure surface and also consider partial submergence, line loadings, and earthquake loadings and presented a new derivation for the Morgenstern-Price method. The proposed derivation is more consistent with that used for the other methods of analysis but utilizes the elements of statics and the assumption proposed by Morgenstern and Price (1965) (Morgenstern and Price,1965:79-93; Morgenstern and Price, 1967:388–393). The Newton-Raphson numerical technique is not used to compute the factor of safety and λ. Griffiths and Lane (1999) (Griffiths and Lane, 1999: 387-403), in this paper a comparison between limit equilibrium method of slices and strength reduction has been done. Chang and Huang(2005)(Chang and Huang,2005:231-240), in this paper a soil slope with 10 m in height was analyzed by limit equilibrium methods and strength reduction.

Limit equilibrium methods of slices:

In the limit equilibrium method of slices we must satisfy critical slip surface, at first. The Factor of Safety (FS) is defined as the ratio of resisting to driving forces on a potential sliding surface. The slope is considered safe only if the calculated safety factor clearly exceeds unity. Most problems in slope stability are statically indeterminate, and as a result, some simplifying assumptions are made in order to determine a unique factor of safety.

Due to the differences in assumptions, various methods have been developed. Among the most popular methods are procedures proposed by Fellenius, Bishop, Janbu, Spencer and Morgenstern-Price's methods referred to before. Some of these methods satisfy only overall moment, like the Ordinary and simplified Bishop Methods and are applicable to a circular slip surface, while Janbu's method satisfies only force equilibrium and is applicable to any shape. Spencer and Morgenstern-Price's methods, however, satisfies both moment and force equilibrium and it is applicable to failure surfaces of any shape. It is considered as one of the rigorous and

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accurate methods for solving stability problems. Table 1 presents a summary of static equilibrium conditions in limit equilibrium methods of slices considered in this study.

Table 1: Static equilibrium conditions in limit equilibrium methods of slices.

Method	Assumption	Failure surface	Equilibrium equation satisfied	Solution by
Swedish method (Fellenius, 1927)	Resultant of interslice force is zero; $J_s = 0$	Circular	Moment	Calculator
Bishop's simplified method (Bishop, 1955)	E_j and E_{j+1} are collinear; $X_j - X_{j+1} = 0$, $J_s = 0$	Circular	Moment	Calculator
Bishop's method (Bishop, 1955)	E_j and E_{j+1} are collinear; $J_s = 0$	Circular	Moment	Calculator/computer
Morgenstern and Price (1965)	Relationship between E and X of the form $X = \lambda f(x)E$; $f(x)$ is a function ≈ 1, λ is a scale factor, $J_s = 0$	Any shape	All	Computer
Spencer (1967)	Interslice forces are parallel; $J_s = 0$	Any shape	All	Computer
Bell's method (Bell, 1968)	Assumed normal stress distribution along failure surface; $J_s = 0$	Any shape	All	Computer
Janbu (1973)	$X_j - X_{j+1}$ replaced by a correction factor, f_o; $J_s = 0$	Noncircular	Horizontal forces	Calculator
Sarma (1975)	Assumed distribution of vertical interslice forces; $J_s = 0$	Any shape	All	Computer

A typical two dimensional slope has been shown in Fig. 1. In this figure resistant and deriving forces have been shown as a sample. In limit equilibrium methods of slices we must divide the upper soil profile in a number slices.

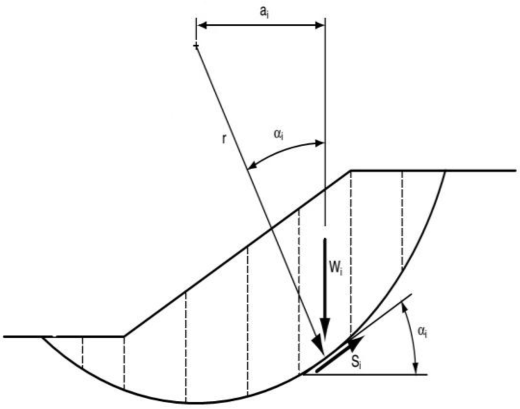

Fig. 1: A typical slope.

By these considerations we can explain limit equilibrium methods of slices as follow.

Ordinary method of slices:

For the Ordinary method of slices (Fellenius, 1936;445-462), which is considered the simplest method of slices, the factor of safety is directly obtained. The method assumes that the inter-slice forces are parallel to the base of each slice, thus they can be neglected and the factor of safety is given as follows:

$$FS = \frac{\sum \left[c^{'}.\Delta l + \left(w.\cos\alpha - u.\Delta l.\cos^2\alpha \right).\tan\varphi^{'} \right]}{\sum w.\sin\alpha} \tag{1}$$

Where:

$w_i = \gamma.b_i.h_i$, $c =$ Cohesion

$\Delta l_i =$ Area of the base of the slice for a slice of unit thickness, $\alpha_i =$ Angle of the base of slice

$w_i =$ Weight of slice, $\gamma =$ Unit weight of soil

U= Pore water pressure

$b_i =$ The width of the slice, $h_i =$ The height of the slice at the centerline

$\varphi =$ Internal friction angle, $FS =$ Factor of safety.

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Simplified Bishop's method:

In Bishop's method (Bishop, 1955:7-17; Duncan,2005,8-12) the factor of safety is determined by trial and errors, using an iterative process, since the factor of safety (FS) appears in both sides of Eq. (2). The inter-slice shear forces are neglected, and only the normal forces are used to define the inter-slice forces. The factor of safety is given as follows:

$$FS=\frac{\sum_{i=1}^{n}\left[\frac{c.\Delta l_i.\cos\alpha_i+(w_i-u.\Delta l.\cos\alpha_i)\tan\varphi}{\cos\alpha_i+(\sin\alpha_i.\tan\varphi)/FS}\right]}{\sum_{i=1}^{n} w_i.\sin\alpha_i} \tag{2}$$

Input parameters were defined as upper.

Simplified Janbu's method:

Similarly, for Janbu's method (Duncan,2005,8-12; Janbu, 1973;47-86) the factor of safety is determined also by an iterative procedure through varying the effective normal stress on the failure surface. The inter slice shear forces are ignored and the normal forces are derived from the summation of vertical forces. The resulting factor of safety is given below:

$$FS=f_0.\left(\frac{\sum_{i=1}^{n}\left[\frac{c.\Delta l_i.\cos\alpha_i+w_i.\tan\varphi}{\cos^2\alpha_i+(\sin\alpha_i.\cos\alpha_i.\tan\varphi)/FS}\right]}{\sum_{i=1}^{n} w_i.\tan\alpha_i}\right) \tag{3}$$

Where:

For $c,\varphi>0$ $\quad f_0=1+0.5\left[\frac{D}{L}-1.4\left(\frac{D}{L}\right)^2\right]$ (4)

Where:

$f_0 =$ Correction factors

$L =$ The length joining the left and right exit points

$D =$ The maximum thickness of the failure zone with reference to this line

Another procedure for f_0 determination.

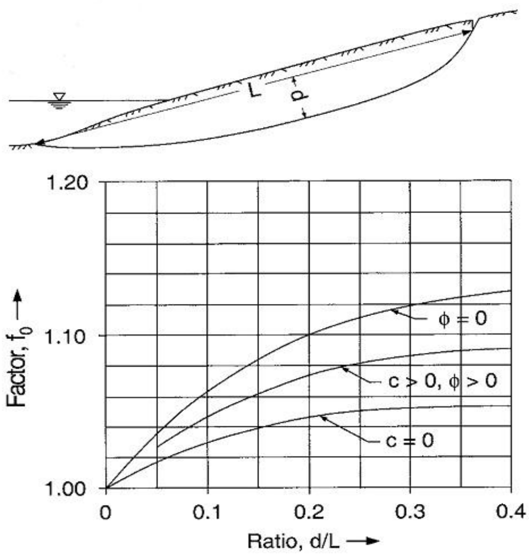

Fig. 2: Determine the f_0

Spencer's method:

In Spencer's method (Spencer, 1967:11-26), the effect of inter-slice forces is included and both moment and force equilibrium are explicitly satisfied. This eventually will lead to an accurate calculation of the factor of safety. The factor of safety is determined through an iterative procedure, slice by slice, by varying FS and θ until force and moment equilibrium are satisfied.

The equation for force equilibrium can be written as:

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$$\sum_{i=1}^{n} Q_i = 0 \tag{5}$$

Where Q_i is the resultant of the interslice forces, and for moment equilibrium, moments can be summed about any arbitrary point. Taking moments about the origin (x=0, y=0) of a Cartesian coordinate system, the equation for moment equilibrium is expressed as:

$$\sum_{i=1}^{n} Q_i.\left(x_{b_i}.\sin\theta - y_{b_i}.\cos\theta\right) = 0 \tag{6}$$

Where x_b is the x (horizontal) coordinate of the center of the base of the slice and y_b is the y (vertical) coordinate of the point on the line of action of the force, Q_i, directly above the center of the base of the slice. Q_i is determined by following equation:

$$Q_i = \frac{w_i.\sin\alpha_i - c.\Delta l_i + w_i.\cos\alpha_i.\tan\varphi / FS}{\cos(\alpha_i - \theta) + \sin(\alpha_i - \theta).\tan\varphi / FS} \tag{7}$$

Where:

$\theta =$ Inter-slice force inclination

Morgenstern-Price's method:

The Morgenstern and Price's procedure(Morgenstern,1963: 121–131; Morgenstern and Price,1967:388–393) assumes that the shear forces between slices are related to the normal forces as:

$$X = \lambda.f(x).E \tag{8}$$

Where X and E are the vertical and horizontal forces between slices, λ is an unknown scaling factor that is solved for as part of the unknowns, and $f(x)$ is an assumed function that has prescribed values at each slice boundary. In the Morgenstern-Price's method, factor of safety is determined by following equation (Zhu DY, 2005: 272–278):

$$FS = \frac{\sum_{i=1}^{n-1}\left(R_i \prod_{j=i}^{n-1} \psi_j\right) + R_n}{\sum_{i=1}^{n-1}\left(T_i \prod_{j=i}^{n-1} \psi_j\right) + T_n} \tag{9}$$

Where R_i is the sum of the shear resistances contributed by all the forces acting on the slices except the normal shear inter-slice forces, and T_i is the sum of the components of these forces tending to cause instability.

Where:

$$\psi_i = \left[(\sin\alpha_{i+1} - \lambda.f_i.\cos\alpha_{i+1}).\tan\varphi + (\cos\alpha_{i+1} + \lambda.f_i.\sin\alpha_{i+1}).FS\right] / \phi_i \tag{10}$$

$$\phi_i = (\sin\alpha_i - \lambda.f_i.\cos\alpha_i).\tan\varphi + (\cos\alpha_i + \lambda.f_i.\sin\alpha_i).FS \tag{11}$$

Procedure:

One of the most important steps in slope stability analysis by limit equilibrium methods of slices is critical slip determination. In this paper for critical slip surface optimization, we use optimization algorithm by SLOPE/W software. For this purpose, a field must be considering for center and another field for radius. In Fig. 3 a flowchart has been represented for Schematic representation of methodology used.

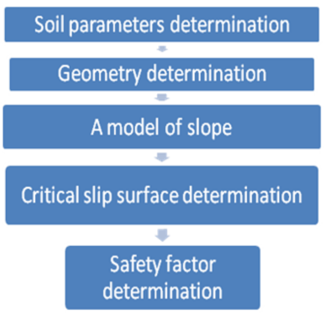

Fig. 3: Schematic representation of methodology.

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Illustrative example:

To examine the accuracy of the methods in determining the safety factor, an example with arbitrary parameter values is demonstrated. A typical slope shape for this example is shown in Fig. 4. The soil parameters are given in Table 2.

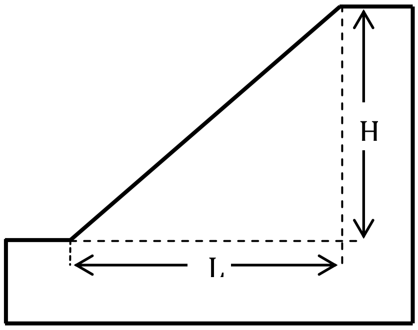

Fig. 4: A typical slope.

Table 2: Parameters in illustrative example.

Parameter	L (m)	H (m)	C (kPa)	Φ (degree)	γ (kN/m^3)
Value	12	12	10	25	20

Comparison:

After definition the slope in SLOPE/W (Figs. 5, 6) and slope stability analysis, output results have been represented in table 3. Critical circular slip surface (Fig. 5) and noncircular slip surface (see Fig. 6) represents.

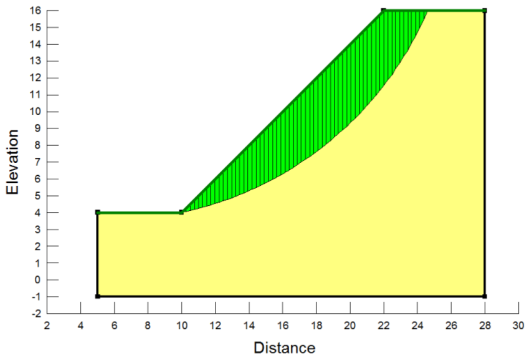

Fig. 5: Circular slip surface.

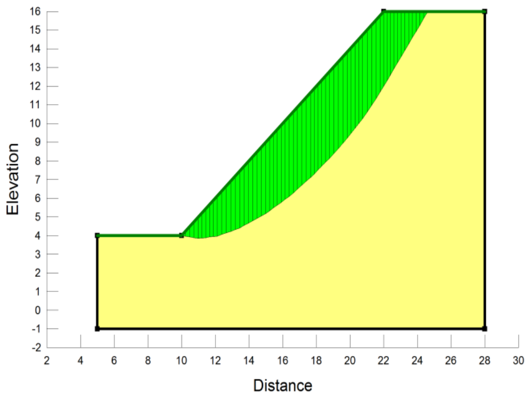

Fig. 6: Noncircular slip surface.

By table 3, minimum safety factor in circular slip surface was obtained by Janbu's method and maximum by Bishop's method. In the noncircular slip surface Ordinary method has minimum and Bishop's method has maximum safety factor. Differences between safety factor in circular and noncircular slip surface are small.

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Table 3: Comparison in Safety factor of different methods.

method	Ordinary	Bishop	Simplified Janbu	Spencer	Morgenstern-Price
Circular slip surface	0.985	1.024	0.950	1.020	1.019
noncircular slip surface	0.963	1.021	0.978	1.015	1.005

A comparison between normal and shear forces in all five methods have been done as follows.

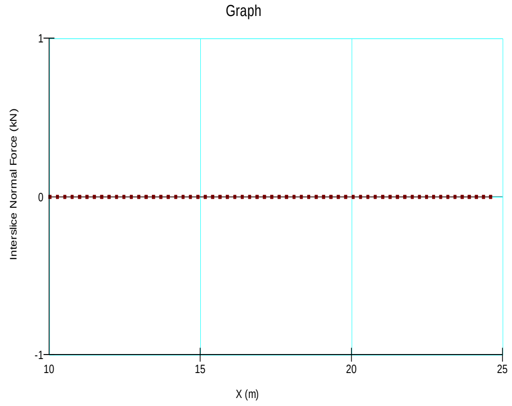

Fig. 7: Inter-slice normal forces in Ordinary method.

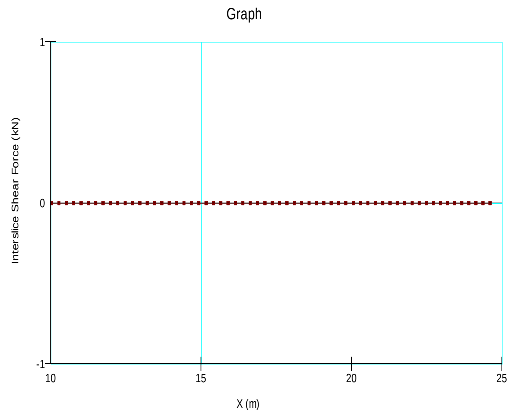

Fig. 8: Inter-slice shear forces in Ordinary method.

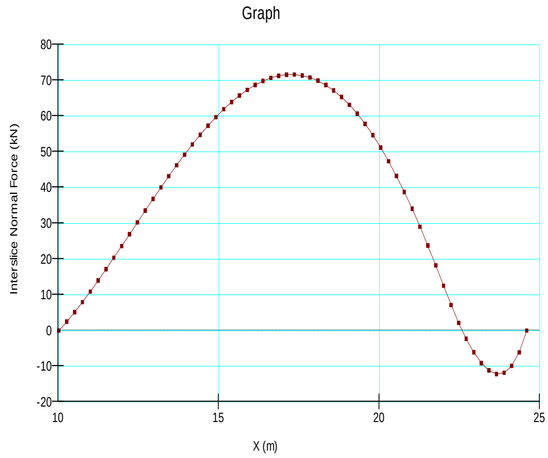

Fig. 9: Inter-slice normal forces in Bishop's method.

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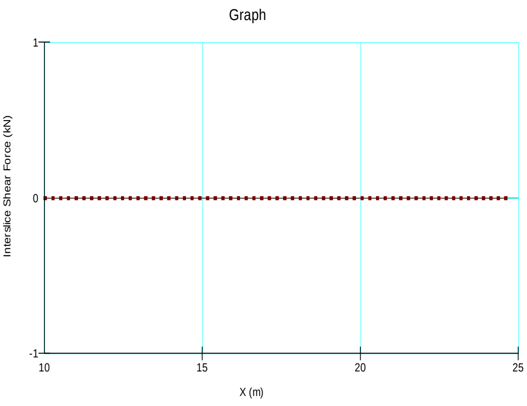

Fig. 10: Inter-slice shear forces in Bishop's method.

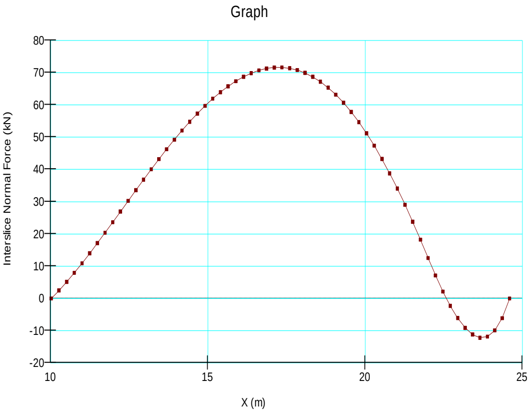

Fig. 11: Inter-slice normal forces in Janbu's method.

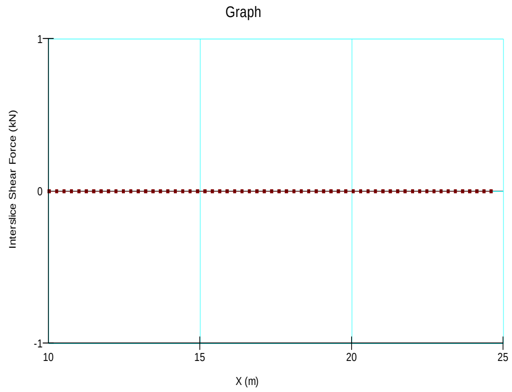

Fig. 12: Inter-slice shear forces in Janbu's method.

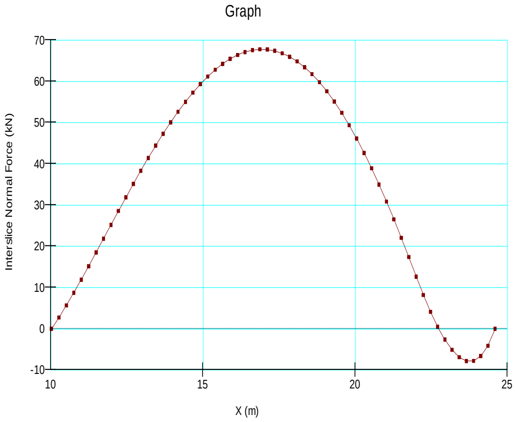

Fig. 13: Inter-slice normal forces in Spencer's method.

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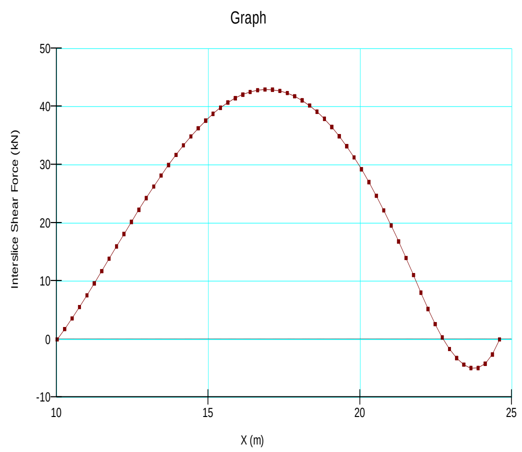

Fig. 14: Inter-slice shear forces in Spencer's method.

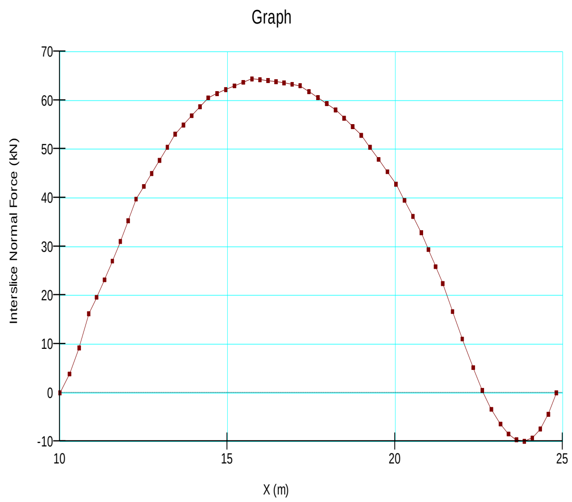

Fig. 15: Inter-slice normal forces in Morgenstern-Price's method.

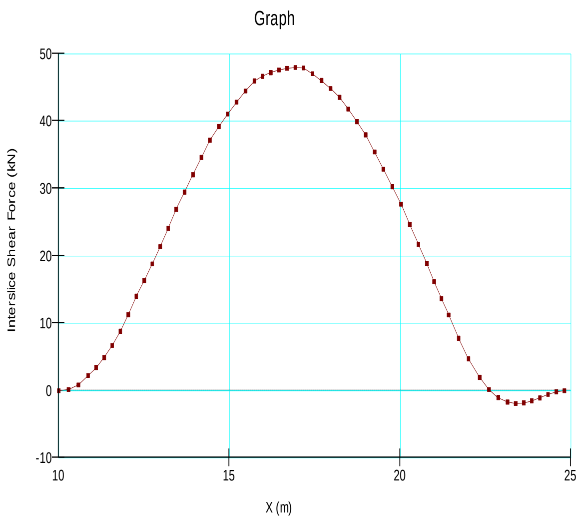

Fig. 16: Inter-slice shear forces in Morgenstern-Price's method.

In Ordinary method, shear and normal forces in all slices are zero (Figs. 7, 8). Inter-slice shear forces also in Bishop and Janbu's methods are zero. Maximum inter-slice normal forces have been obtained in Bishop and Janbu's methods. Maximum inter-slice shear forces have been determined in Morgenstern-Price's method.

Shear mobilized forces for any slice represents in Fig. 17. Mobilized forces in Janbu's method is maximum and in Morgenstern-Price's method is minimum (Fig. 17).

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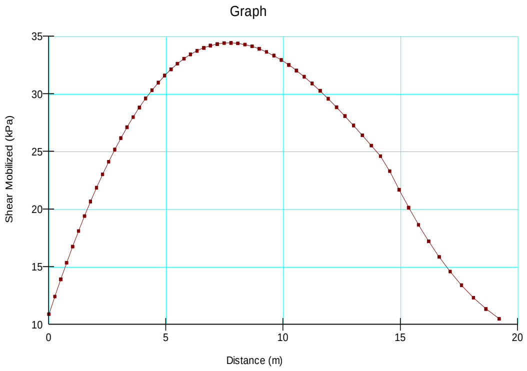
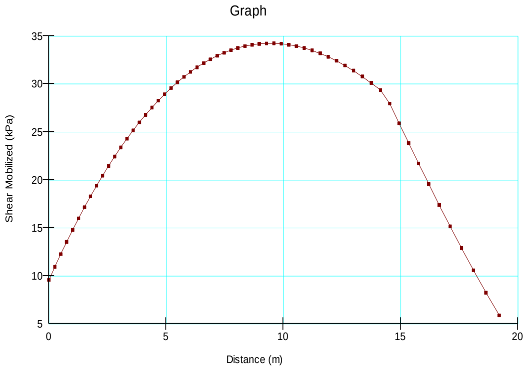
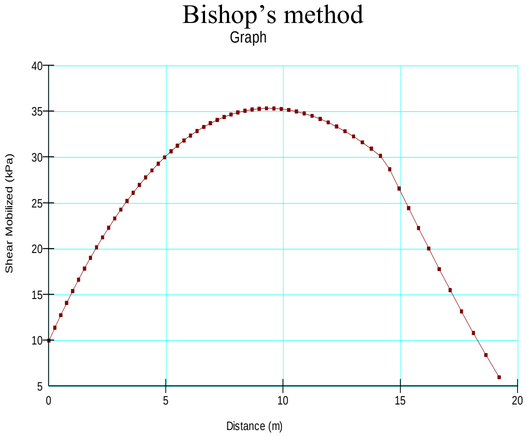


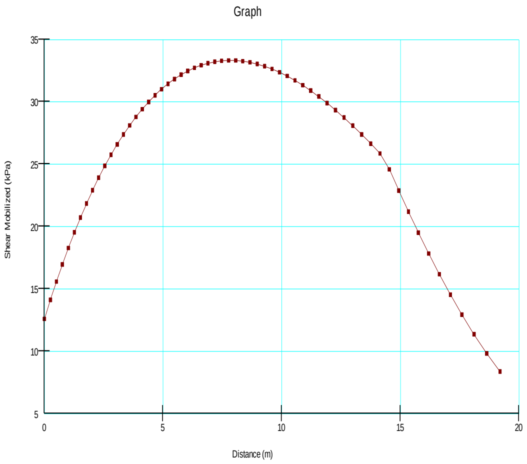
Ordinary method



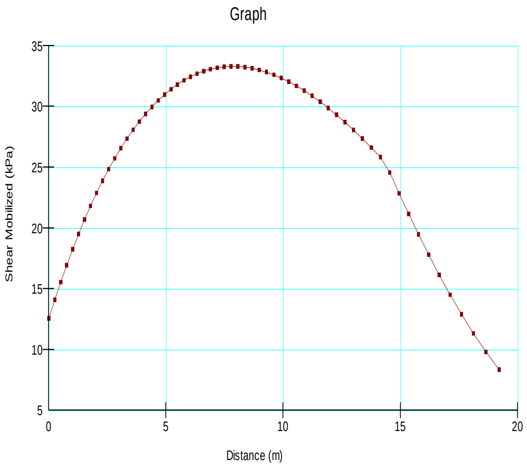
Bishop's method



Simplified Janbu's method



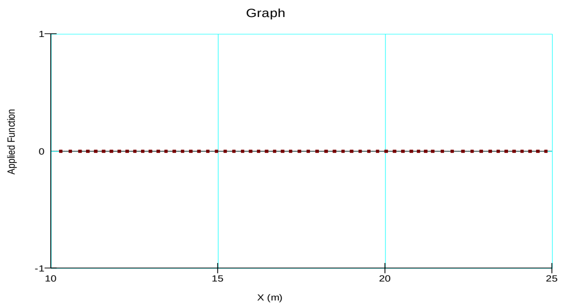
Spencer's method



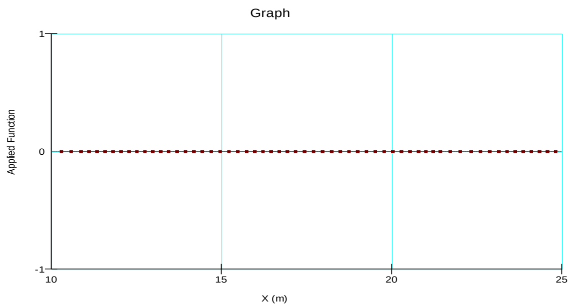
Moegenstern-Price's method

Fig. 17: Mobilized forces in varoious methods.

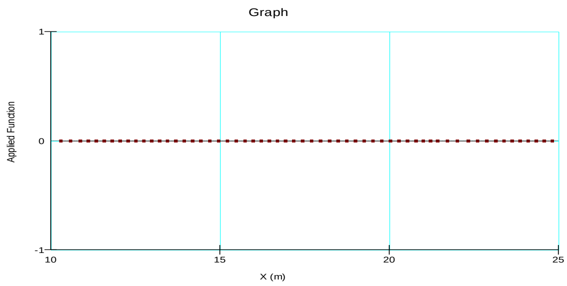
One of the differences in limit equilibrium methods of slices is differ in inter-slice forces inclination. This difference has been presented in Fig. 18. This inclination is zero for Ordinary, Bishop and Janbu's methods. The inter-slice forces inclinations is equal for all slices In Spencer's method and For Morgenstern-Price's method inter-slice inclinations defines by Eq. (8) with a sinusoidal function.



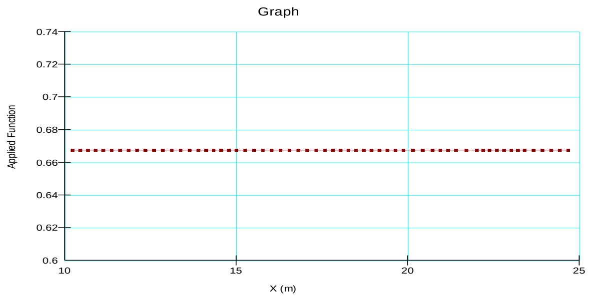
Ordinary method



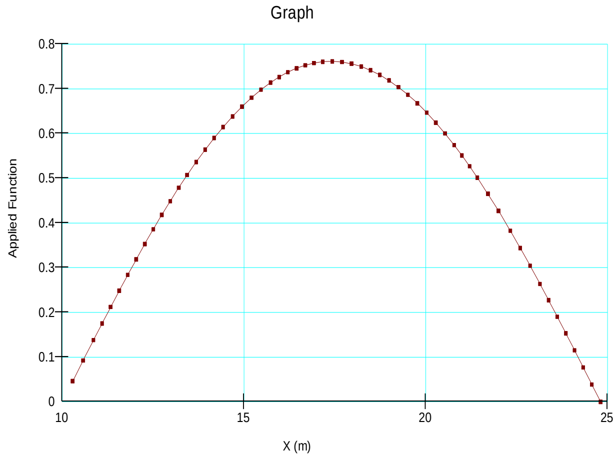
Bishop's method



Janbu's method



Spencer's method



Morgenstern-Price's method

Fig. 18: Inclination of inter-slice forces.

Conclusion:

In this paper, we tried to compare limit equilibrium methods of slices and compare the influences of the methods assumptions in results. Because of, any method has different assumptions. Minimum safety factor in circular slip surface was obtained by Janbu's method and maximum by Bishop's method. In the noncircular slip surface Ordinary method has minimum and Bishop's method has maximum safety factor. Differences between safety factor in circular and noncircular slip surface are small.

Morgenstern-Price and Spencer's methods are and their assumptions are most logical than other methods. Therefore, results of these methods are most suitable.

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