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Analytic Hierarchy Process: Obtaining weight vector with generalized weighted least square method by using Genetic Algorithm and simplex method

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Background: Analytic Hierarchy Process (AHP) is a famous technique to determine priority of alternatives in a decision making process. The basic version of AHP has a variety of generalizations and extensions, making it more useful and more appropriate for solving real world problems. An important issue in AHP is finding a useful method or model that can determine a true weight vector for pairwise comparison matrices. Weighted least-square method (WLSM) is a method that can be generalized and it is useful for consistent an inconsistent matrices. **Objective:** To generalize WLSM, two models and an approach is proposed and Genetic Algorithm (GA) and simplex method is used to solve them. **Results:** Compared with conventional techniques, the proposed models have least computations. Because GA in MATLAB 9.1 is used for one of models and GA and simplex method are used for second model. Different norms are applied in compare with common WLSM. It can produce a true weight vector for pairwise comparison matrices. We can rank the alternatives in AHP by obtained weight vector. **Conclusion:** We have proposed two models according the common WLSM and called them generalized WLSM models. They are in two kinds: Nonlinear and Linear. GA and Simplex method are used for solving them. When Compared with conventional techniques, the proposed models and used solvers reduce computational complexity

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INTRODUCTION

Multi-criteria decision making with many models and applications is an important part of operations research (Belton and Stewart, 2002). Multi-criteria decision making models are well developed and have been employed in many applications (Ananda and Herath, 2009; Diaz-Balteiro and Romero, 2008; Weintraub and Romero., 2007). Analytic hierarchy process (AHP) (Saaty,1980) a well-known approach for handling multi-criteria decision making problems, is suitable for solving multi-criteria problems, evaluating and ranking alternatives, and supporting the selection of the best alternative. AHP is very flexible since it enables combining empirical data and subjective judgments, and intangible and immeasurable criteria. It enables us to handle the complexity of the real world multicriteria problems. AHP is based on a hierarchical structure of criteria, sub-criteria, and alternatives. Its foundations are pair-wise comparisons of objects (criteria, sub-criteria, alternatives) on the same level regarding the object on the next higher level. Pair-wise comparisons are gathered in a comparison matrix. The priority vectors are derived from the comparison matrices by one of the known methods and synthesized in the final priority vector. AHP has been applied in numerous fields (Vaidya in Kumar, 2006).

The basic version of AHP has a variety of generalizations and extensions, making it more useful and more appropriate for solving real world problems: extending hierarchical structure to network, replacing crisp judgments with interval judgments, and extending one decision maker to group decision making. Although the method has been widely employed, some open questions remain.

There are different applications of AHP. For the convenience, these applications have been classified into three groups, namely: (a) applications based on a theme, (b) specific applications, and (c) applications combined with some other methodology.

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In this paper, we focus on pairwise comparison matrices and obtaining weight vector for it. For this purpose, two methods are used. Algorithm Genetic(GA) and simplex method is used for solving them with a special approach.

The format of this paper is as follows: In Section 2, we review weighted least square method (WLSM) and Genetic Algorithm (GA). In Section 3, we present a modified weighted least square method (WLSM) for determining the priorities of alternatives with obtained weight vector. For solving the two models, GA and simplex method are used. In Section 4, we use a numerical example to illustrate the proposed method in this study. Finally, we provide some concluding remarks and suggestions for future work in Section 5.

Methodology:

2. A review on WLSM and GA:

2.1 A review on weighted least square Method (WLSM):

Chu et al. (1979) introduced weighted least square method (WLSM) to determine weight vector. Basis of this method is consistency and inconsistency of a pairwise comparison matrix.

Let $A=(a_{ij})_{n\times n}$ be a pairwise comparison matrix with $a_{ii}=1$ and $a_{ji}=\frac{1}{a_{ij}>0}$ for $i\neq j$ and $W=(w_1,\dots,w_n)^T$ be its local priority vector. Consider the elements a_{ij} of Saaty's matrix *A* in $AW=\lambda_{\max}W$. They desire to determine the weights w_i , such that, given a_{ij} ,

$$a_{ij}\approx\frac{w_i}{w_j} \tag{1}$$

The weights can be obtained by solving following model:

$$\begin{aligned} \min \quad & \sum_{i=}^{n}\sum_{j=1}^{n}\left(a_{ij}-\frac{w_i}{w_j}\right)^2 \\ s.t \quad & \sum_{i=1}^{n}w_i=1 \\ & w_i\geq 0 \qquad i=1,\dots,n. \end{aligned} \tag{2}$$

Their model is a nonlinear programming that minimizes the Euclidian distance between the ideal and actual solution. It is much more difficult to solve numerically. In order to solve it, following model is solved.

$$\begin{aligned} \min \quad & \sum_{i=}^{n}\sum_{j=1}^{n}\left(a_{ij}w_j-w_i\right)^2 \\ s.t \quad & \sum_{i=1}^{n}w_i=1 \\ & w_i\geq 0 \qquad i=1,\dots,n. \end{aligned} \tag{3}$$

This method is conceptually easier to understand than the eigenvector method. Moreover, results of this method have showed that the w_i 's are greater than zero. Indeed, they used lagrange multipliers to solve model (1), and convert it to a linear equations system that is solved easily.

We generalize models (1) and (2) to a general cases and use GA and simplex method to solve generalized models.

2.2 Genetic Algorithm (GA):

There is a great need for powerful heuristics that find good suboptimal solutions in reasonable amounts of computing time. These algorithms are usually very simple and have short running times. There are a huge number of papers dealing with finding near optimal solution for most of the problems. Our aim is to present the most interesting and efficient algorithms.

When a heuristic is designed, the question, which arises, is about the quality of the produced solution. There are three different ways that one may try to answer this question, namely; (a) Empirical, (b) Worst case analysis, (c) Probabilistic analysis.

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Genetic Algorithms (GAs) are search procedures based on the mechanics of natural selection and natural genetics. The first GA was developed by John H. Holland in the 1960s to allow computers to evolve solutions to difficult search and combinatorial problems, such as function optimization and machine learning (Holland, 1975).

Genetic algorithms offer a particularly attractive approach for problems. They are generally quite effective for rapid global search of large, non-linear and poorly understood spaces. Moreover, genetic algorithms are very effective in solving large scale problems. GAs are based on an imitation of the biological process in which new and better populations among different species are developed during evolution. A GA is a stochastic iterative procedure that maintains the population size constant in iteration, called a generation. Their basic operation is the mating of two solutions in order to form a new solution. To form a new population, a binary operator called crossover, and a unary operator, called mutation, are applied (Reeves, 1995; Reeves, 2003). Crossover takes two individuals, called parents, and produces two new individuals, called offsprings, by swapping parts of the parents.

Results:

In this section, we generalize models (2) and (3). Indeed, we consider different norms in objective functions. Then use GA and simplex method to solve given models with different norms.

The first proposed model is as following:

$$\begin{aligned} \min \quad & \left(\sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{w_i}{w_j} - a_{ij} \right)^p \right)^{\frac{1}{p}} \\ s.t \quad & \sum_{j=1}^{n} w_j = 1 \\ & w_j \geq 0 \qquad j = 1, \ldots, n. \end{aligned} \tag{4}$$

where a_{ij} are elements of a pairwise comparison matrix. In this model, norm 1, norm two and infinite norm are used to minimize the distance between the ideal and actual solution. An approach beside these norms is proposed. The approach is that, in first stage with a chosen norm, model is solved and weights are obtained, the best one is omitted and rank one belongs to deleted one. Therefore, a row and a column corresponding to it are deleted. Second stage, the model is solved for (n-1) alternatives, and optimal weights are obtained, the best weight is omitted and rank two is belonged to it. This process is repeated until one alternative is remained.

The alternatives are ranked with this approach after using model (1).

We substitute p=1 and p=2 in model (4) for norm 1 and norm 2 respectively. Indeed, P=2 is the same model (2).

In infinite norm, the model (4) is converted to following model:

$$\begin{aligned} \min \ \max \ & \left\{ \left| \frac{w_i}{w_j} - a_{ij} \right| \right\}_{i,j=1,\ldots,n} \\ s.t \quad & \\ & w_j \geq 1 \qquad j = 1, \ldots, n \end{aligned} \tag{5}$$

In consistent pairwise comparison matrix , we have:

$$a_{ij} = \frac{w_i}{w_j} \tag{6}$$

Assume that there is consistent pairwise comparison matrix , the optimal value of objective function will be zero according to (6). For solving model (4) with norm one, norm two and model (5), GA is used. In model (5), the constraint is considered as $w_j \geq 1$ to keep away the w_j from zero.

The second proposed model is as following:

$$\begin{aligned} \min \quad & \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\left(a_{ij}w_j - w_i\right)^p\right)^{1/p} \\ s.t \quad & \sum_{i=1}^{n} w_i = 1 \\ & w_i \geq 0 \qquad i = 1,\dots,n. \end{aligned} \tag{7}$$

We call this model general modified WLSM. We substitute p=1 and p=2 in model (7) for norm 1 and norm 2 respectively. We verify each case separately. Indeed, P=2 is the same model (3). Therefore, we can use proposed approach with GA. For p=1, the model can rewrite as following:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\left|a_{ij}w_j - w_i\right| \\ s.t \quad & \sum_{i=1}^{n} w_i = 1 \\ & w_i \geq 0 \qquad i = 1,\dots,n. \end{aligned} \tag{8}$$

In this case, model (8), $a_{ij}w_j - w_{ij} = \overset{+}{u}_{ij} - \overset{-}{u}_{ij}$ is used which $\overset{+}{u}_{ij}\overset{-}{u}_{ij} = 0$, $\overset{+}{u}_{ij}$, $\overset{-}{u}_{ij} \geq 0$ to transform model (8) to a linear programming. Therefore, model is equivalent to following model:

$$\begin{aligned} \min \quad & \sum_{i=1}^{n}\sum_{j=1}^{n}\overset{+}{u}_{ij} + \overset{-}{u}_{ij} \\ s.t \quad & \sum_{i=1}^{n} w_i = 1 \\ & a_{ij}w_j - w_i - \overset{+}{u}_{ij} + \overset{-}{u}_{ij} = 0 \qquad i,j = 1,\dots,n \\ & w_i \geq 0 \qquad i = 1,\dots,n \\ & \overset{+}{u}_{ij}, \overset{-}{u}_{ij} \geq 0 \quad i,j = 1,\dots,n. \end{aligned} \tag{9}$$

For solving this, we apply MATLAB 9.1 software, simplex method, optimal solutions are obtained, and then approach is used to ranking alternatives in AHP.

For infinite norm in model (7), the following model is written.

$$\begin{aligned} \min\ & \max\{\left|a_{ij}w_j - w_i\right|\}_{i,j=1,\dots,n} \\ s.t \quad & \sum_{i=1}^{n} w_i = 1 \\ & w_i \geq 0 \qquad i = 1,\dots,n. \end{aligned} \tag{10}$$

We try to make it simple to solve and have least computations. Consequently, we put $\max\left\{\left|a_{ij}w_j - w_i\right|\right\}_{i,j=1,\dots n} = z$ and model (7) is rewriting as following:

$$\begin{aligned} \min \quad & z \\ s.t \quad & a_{ij}w_j - w_i \leq z \\ & a_{ij}w_j - w_i \geq -z \\ & \sum_{i=1}^{n} w_i = 1 \\ & z \geq 0 \quad w_i \geq 0 \qquad i = 1,\dots,n. \end{aligned} \tag{11}$$

For solving this model, simplex method with MATLAB 9.1 and proposed approach are used. According this method, alternatives are ranked.

Up to here, we solve difficulty of computing by GA and proposed approach. Moreover, theses models have no drawback, when there are alternative solutions.

As it is claimed, the GA is applied as the solver. The characteristics of the applied GA are as following. The size of population can be changed to get exact or almost true solutions.

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Table 1: Characteristics of GA

Crossover	heuristic, rate 2
Population Size	850
Selection	tournament, rate 8
Crossover Fraction	0.8
Mutation	adapt feasible
Generations	500
Rate of tolerance	1.0000e-010
Migration	0.2

The mathematical details of the proposed steps are presented below:

Step 1: Calculate the optimal value of objective function and w_i of model (4) and (7) using different norms and omitted approach.

Step 2: For solving models with different norms and omitted approach, GA and simplex method are used. Therefore, alternatives are ranked.

The procedure of the proposed method is illustrated in numerical example.

Discussion:

Consider the following inconsistent comparison matrix, which comes from Saaty (2000).

$$A = \begin{bmatrix} 1 & 4 & 3 & 1 & 3 & 4 \\ \frac{1}{4} & 1 & 7 & 3 & \frac{1}{5} & 1 \\ \frac{1}{3} & \frac{1}{7} & 1 & \frac{1}{5} & \frac{1}{5} & \frac{1}{6} \\ 1 & \frac{1}{3} & 5 & 1 & 1 & \frac{1}{3} \\ \frac{1}{3} & 5 & 5 & 1 & 1 & 3 \\ \frac{1}{4} & 1 & 6 & 3 & \frac{1}{3} & 1 \end{bmatrix}$$

Model (4) is used for given matrix. Tables 2, 3 and 4 report the result of using GA and approach for solving the model (4).

Table 2: Result for alternatives with regards to norm 1, GA and Approach

Alternatives	1	2	3	4	5	6	Best
First stage	0.511	0.128	0.021	0.106	0.106	0.128	Alternative 1
Second stage		0.260	0.043	0.217	0.217	0.261	Alternative 6
Third stage		0.394	0.055	0.276	0.276		Alternative 2
Fourth stage			0.091	0.454	0.455		Alternative 5
Fifth stage			0.167	0. 833			Alternative 4
Sixth stage			1				Alternative 3

Table 3: Result for alternatives with regards to norm 2, GA and Approach

Alternatives	1	2	3	4	5	6	Best
First stage	0.363	0.077	0.034	0.136	0.201	0.189	Alternative 1
Second stage		0.121	0.053	0.211	0.317	0.298	Alternative 5
Third stage		0.394	0.051	0.228		0.326	Alternative 2
Fourth stage			0.084	0. 380		0.536	Alternative 6
Fifth stage			0.167	0. 833			Alternative 4
Sixth stage			1				Alternative 3

Table 4: Result for alternatives with regards to infinite norm, GA and Approach

Alternatives	1	2	3	4	5	6	Best
First stage	0.55	0.05	0.03	0.09	0.17	0.12	Alternative 1
Second stage		0.11	0.06	0.19	0.37	0.28	Alternative 5
Third stage		0.96	0.00	0.01		0.02	Alternative 2
Forth stage			0.08	0.32		0.60	Alternative 6
Fifth stage			0.17	0. 83			Alternative 4
Sixth stage			1				Alternative 3

In each Table, after applying model (4) with mentioned method and solver in stage one, optimal weights are achieved. According the obtained weights, the best one gets rank 1 and delete from set of alternatives. In stage 2, model (4) is used for 5 alternatives with the process that is described. The best weight gets rank 2 and deletes.

This process is continued until one alternative is remained. In this way, ranks of all alternatives are determined. Final ranking of alternatives is summarized as below:

Table 5: Final ranking using three norms in model (4)

	Alternative 1	Alternative 2	Alternative 3	Alternative 4	Alternative 5	Alternative 6
Ranking by norm 1	1	3	6	5	4	2
Ranking by norm 2	1	3	6	5	2	4
Ranking by infinite norm	1	3	6	5	2	4

As Table 5 shows, our result is true and consistent with most of methods that researchers are agreed with them. Moreover, our model can determine a weight vector that is consistent with matrix A.

In following, model (7) is used to obtain weight vector. During calculations, necessary transformations for different norms are done. Indeed, models (3), (8) and (10) are applied. For solving this model with norms 1 and infinite, simplex method and approach are used and solving it with norm 2, GA and approach is used. The result of using model (3), are emerged in Tables (6), (7) and (8).

Table 6: Result for alternatives with regards to norm 1, simplex method and Approach

Alternatives	1	2	3	4	5	6	Best
First stage	0.48	0.12	0.02	0.10	0.16	0.12	Alternative 1
Second stage		0.1818	0.0303	0.0606	0.5455	0.1818	Alternative 5
Third stage		0.4038	0.0577	0.1436		0.4038	Alternative 2
Forth stage			0.1111	0.2222		0.6666	Alternative 6
Fifth stage			0.1666	0.8333			Alternative 4
Sixth stage			1				Alternative 3

Table 7: Result for alternatives with regards to norm 1, GA and Approach

Alternatives	1	2	3	4	5	6	Best
First stage	0.33	0.06	0.04	0.12	0.23	0.22	Alternative 1
Second stage		0.09	0.05	0.15	0.39	0.31	Alternative 5
Third stage		0.41	0.05	0.14		0.39	Alternative 2
Forth stage			0.09	0.25		0.66	Alternative 6
Fifth stage			0.02	0. 83			Alternative 4
Sixth stage			1				Alternative 3

Table 8: Result for alternatives with regards to norm 1, simplex method and Approach

Alternatives	1	2	3	4	5	6	Best
First stage	0.4478	0.1119	0.0299	0.1439	0.1439	0.1119	Alternative 1
Second stage		0.0667	0.0667	0.133	0.333	0.4	Alternative 6
Third stage		0.1544	0.055	0.0515	0.7721		Alternative 5
Forth stage		0.6774	0.0968	0.02258			Alternative 2
Fifth stage			0.1666	0.8333			Alternative 4
Sixth stage			1				Alternative 3

Final ranking of three norms in model (7) is reported in Table 9. In Tables 6, 7 and 8

We use model (7). Some necessary transformations are done in model (7) with regard to chosen norm. Then, in each stage, maximum value between obtained weights is omitted and is chosen as a best one.

Table 9: Final ranking using three norms in model (7)

	Alternative 1	Alternative 2	Alternative 3	Alternative 4	Alternative 5	Alternative 6
Ranking by norm 1	1	3	6	5	2	4
Ranking by norm 2	1	3	6	5	2	4
Ranking by infinite norm	1	4	6	5	3	2

In general with both Models (4) and (7), we can claim directly that following ranking is concluded for matrix *A*.

Table 10: Final Ranking for matrix *A*

Alternatives	Rank
Alternative 1	1
Alternative 5	2
Alternative 2	3
Alternative 6	4
Alternative 4	5
Alternative 3	6

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This ranking is consistent with the information of pairwise comparison matrix *A*. With respect to the first row of matrix *A*, it is obvious that C_1 is more important than others, and can be removed from pairwise comparison matrix. In this reduced matrix, C_5 is the most important alternative or criterion and should be ranked in the second place. Therefore, it can be eliminated from the matrix for further comparisons. From this reduced pairwise comparison matrix, it is observed that C_2 is weakly more important than C_6 because it is seven times as important as C_3, while C_6 is only six times as important as C_3. Therefore, C_2 and C_6 are, respectively, the third and fourth most important alternatives or criteria. The left criteria or alternatives, C_3 and C_4, are very easy to be ranked as $C_4 > C_3$ in terms of their direct comparisons $a_{34} = \frac{1}{5}$ and $a_{43} = 5.$ Therefore, the ranking obtained by the our proposed model is consistent with pairwise comparison matrix information. This shows that models (4) and (7) can produce logical local weights for inconsistent pairwise comparison matrices.

Conclusion:

In this paper, we reviewed weighted least square method in nonlinear and linear form. This model is extended to general case. To solve generalized models, Genetic Algorithm (GA), simplex method and an interesting approach are used. The example demonstrated that they provide similar results as the well known eigenvector method (EM).Theirs great benefit is their formulation. These models produced rational weights for inconsistent matrices. In this research, it was assumed that the data of the pairwise comparisons were deterministic. Future researches can be conducted by using the general WLSM with fuzzy and stochastic data.

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