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A Slacks-Based Efficiency Measure for Modelling Environmental Performance

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ABSTRACT

Data Envelopment Analysis (DEA) is a well-established nonparametric methodology concerned with evaluating the performances of the firms with multiple inputs and outputs. However, an important issue is that the firms in the real world unavoidably generate undesirable outputs (like pollutants) along with desirable outputs. Therefore, the undesirable outputs should be incorporated into the performance assessment. This study develops a slacks-based efficiency measure for modelling undesirable outputs into the environmental performance. After adjusting the undesirable outputs to their minimum levels, the proposed measure considers all the input excesses and desirable output shortfalls in a standardized efficiency score. Then, the presented method is applied to study the pollutants (waste gas emission and waste discharge) of 31 administrative regions of China. From this application, it is concluded that 7 industries pay attention to the reduction of their pollutants accompanying with improving their commercial targets.

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INTRODUCTION

Data Envelopment Analysis (DEA) is a well-established nonparametric methodology concerned with evaluating the performance of the Decision Making Units (DMUs) with multiple inputs and outputs. However, an important issue is that the DMUs in the real world unavoidably generate undesirable outputs (like pollutants) along with desirable outputs. Nowadays, global warming, climate change, an enhanced emission of CO_2 in the air, and water pollution are major problems all over the world. These worldwide problems clarify the importance of incorporating undesirable outputs into performance assessment.

Recently, many researchers try to model undesirable outputs within the DEA framework. Dealing with this topic, Fare *et al*. (1989) proposed an approach in 1989. They replaced the disposability of outputs with weak disposability. Later, the study in this direction was extended by many researchers such as Fare *et al*. (1993, 2005), Scheel (2001), Hailu and Veeman (2001), Kuosmanen (2005), and Zhou *et al*. (2007).

Moreover, a common treatment of undesirable outputs is to think of them as inputs. Some of the works in this area include: Dyson *et al*. (2001), Seiford and Zhu (2002), Dyckhoff and Allen (2001), and Sueyoshi and Goto (2011). Nevertheless, this procedure causes to be concerned with two problems. First, the free disposability principle between inputs and bad outputs implies that a finite amount of input can produce an infinite amount of undesirable outputs, while this is physically impossible (Fare and Grosskopf, 2003). Second, the free disposability principle does not recognize the relation between the desirable and undesirable outputs. Consequently, the undesirable outputs should model as outputs and the link between desirable and undesirable outputs should be taken into account in performance evaluation. Shephard (1974) was the first to introduce the weak disposability principle between good and bad outputs. Based on this principle, he presented a technology dealing with both good and bad outputs. Later, Kuosmanen (2005) followed this way and offered another technology that was more flexible. Then, Kuosmanen and Podinovski (2009) examined that the Shephard technology suffers from some drawbacks, and its serious drawback is that it is not convex. In addition, they demonstrated that the Kuosmanen technology is the only correct technology suitable for modelling undesirable outputs under weak disposability (2009). The main purpose of this paper is to provide a slacks-based efficiency measure for modelling environmental performance. This measure is obtained by solving some DEA-based models. First, the undesirable factors are adjusted to their minimum levels, and then, all the input excesses and desirable output shortfalls are computed.

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The rest of this paper is arranged as follows: In section 2, we briefly introduce the two technologies developed by Shephard (1974) and Kuosmanen (2005) for modelling undesirable factors. Section 3 focuses on Kuosmanen technology and provides a slacks-based measure of efficiency for evaluating the environmental performance of DMUs. In section 4, we apply the proposed measure to study the pollutants (waste gas emission ad waste discharge) of 31 administrative regions of China. Summary and conclusions of the study are given in section 5.

Preliminaries:

In this paper, it is supposed that there are *n* observed DMUs (Decision Making Units) and the *j*th DMU, $j \in \{1,...,n\}$, is determined by the vector (x_j, g_j, b_j), where $x_j = (x_{1j}, x_{2j}, ..., x_{mj}) \in R^m, x_j \geq 0, x_j \neq 0$ is the vector of inputs, $g_j = (g_{1j}, g_{2j}, ..., g_{sj}) \in R^s, g_j \geq 0, g_j \neq 0$ is the vector of desirable (good) outputs and $b_j = (b_{1j}, b_{2j}, ..., b_{hj}) \in R^h, b_j \geq 0, b_j \neq 0$ is the vector of undesirable (bad) outputs. The production technology is characterized by the set $\mathrm{T} = \{(x, g, b) \mid \text{x can produce } (g, b)\}$. Consider the following principles which have been introduced in the DEA literature (Shepard, 1974; Kousmanen, 2005) for incorporating undesirable factors into production technology:

A1 Strong (free) disposability of inputs and good outputs. If $(x, g, b) \in T, 0 \leq g' \leq g$ and $x' \geq x$, then $(x', g', b) \in T$.
A2 Weak disposability of good and bad outputs. If $(x, g, b) \in T, 0 \leq \theta \leq 1$, then $(x, \theta g, \theta b) \in T$.
A3 T is convex.

Axiom (A2) recognizes the relation between good and bad outputs, because the pollutants can be reduced in proportion to the reduction of good outputs. The multiplier θ used in this axiom is pointed out as the abatement factor (Kousmanen, 2005).

Shephard (1974) applied a single abatement factor to model weak disposability and presented the following technology T_S:

$$T_S = \{(x, g, b) \mid \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \theta \lambda_j g_j \geq g, \sum_{j=1}^{n} \theta \lambda_j b_j = b, \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, (j = 1, ..., n), 0 \leq \theta \leq 1\} \tag{1}$$

Note that variables $\lambda_1, \lambda_2, ..., \lambda_n$ are the structural variables.

Later, Kuosmanen (2005) examined an alternative approach to deal with axiom (A2). He argued that the correct minimum extrapolation technology necessitates *n* distinctive abatement factors. Therefore, he employed distinctive abatement factors $\theta_j, (j = 1, ..., n)$ corresponding to each observed firm, and developed the following technology T_K:

$$T_K = \{(x, g, b) \mid \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \theta_j \lambda_j g_j \geq g, \sum_{j=1}^{n} \theta_j \lambda_j b_j = b, \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, (j = 1, ..., n), 0 \leq \theta_j \leq 1, (j = 1, ..., n)\} \tag{2}$$

Subsequently, Kuosmanen and Podinovski (2009) claimed that the Kuosmanen approach is more adaptable with respect to the choice of abatement factors. Moreover, using a simple numerical example, they showed that the Shephard technology is not convex; rather the Kuosmanen technology is convex. Furthermore, they proved that T_K is indeed the correct technology that satisfies the minimum extrapolation principle of DEA under the mentioned axioms of (A1), (A2) and (A3).

Additionally, it should be noted that T_S and T_K are both nonlinear, since θ and θ_j are multiplied with λ_j. Nevertheless, Kuosmanen (2005) stated that T_S can be linearized as follows:

$$\lambda_j = \underbrace{\theta_j \lambda_j}_{\eta_j} + \underbrace{(1 - \theta_j \lambda_j)}_{\mu_j}, \quad (j = 1, ..., n).$$

$$T_K = \{(x, g, b) \mid \sum_{j=1}^{n} (\eta_j + \mu_j) x_j \leq x, \sum_{j=1}^{n} \eta_j g_j \geq g, \sum_{j=1}^{n} \eta_j b_j = b, \\ \sum_{j=1}^{n} (\eta_j + \mu_j) = 1, \eta_j \geq 0, \mu_j \geq 0, (j = 1, ..., n)\} \tag{3}$$

However, Kuosmanen and Podinovski emphasized that T_S cannot be linearized by the above approach, because it is not convex (2009).

The Proposed Slacks-Based Efficiency Measure:

This section attempts to present a slacks-based measure that incorporates both desirable and undesirable outputs. To this end, the Kuosmanen technology is applied, because its usefulness and strengths were clarified in the previous section.

The proposed approach proceeds in two stages. In stage 1, the priority is accorded to the undesirable outputs and they are minimized as much as possible. The yielded measure in this stage is denoted by *EI* (Environmental Index). This index identifies all the inefficiencies which arise from undesirable factors. Indeed, *EI* estimates the capability of firms in managing pollutants. Therefore, the following model is employed:

$$
\begin{aligned}
EI = Min \quad & \frac{1}{h}\sum_{f=1}^{h}\gamma_f \\
s.t. \quad & (x_{1k},\dots,x_{mk},g_{1k},\dots,g_{sk},\gamma_1 b_{1k},\dots,\gamma_h b_{hk})^t \in T_K \\
& 0 \le \gamma_f \le 1, \quad (f = 1,\dots,h
\end{aligned}
\tag{4}
$$

Here, k indicates the DMU under evaluation. The constraints $0 \le \gamma_f \le 1, (f = 1,\dots,h)$ are included into model (4) to see whether a DMU can be found to dominate DMU_k in undesirable factors. In this model, if $b_{fk} = 0$, then the term γ_f is deleted from the objective function. Using T_K, the outcome model is:

$$
\begin{aligned}
EI = Min \quad & \frac{1}{h}\sum_{f=1}^{h}\gamma_f \\
s.t. \quad & \sum_{j=1}^{n}(\eta_j + \mu_j)x_{ij} \le x_{ik} \qquad (i = 1,\dots,m), \\
& \sum_{j=1}^{n}\eta_j g_{rj} \ge g_{rk} \qquad (r = 1,\dots,s), \\
& \sum_{j=1}^{n}\eta_j b_{fj} = \gamma_f\, b_{fk} \qquad (f = 1,\dots,h), \\
& \sum_{j=1}^{n}(\eta_j + \mu_j) = 1, \eta_j \ge 0, \mu_j \ge 0 \quad (j = 1,\dots,n), \\
& 0 \le \gamma_f \le 1 \quad (f = 1,\dots,h).
\end{aligned}
\tag{5}
$$

Stage 2 considers all the input excesses and good output shortfalls in a standardized efficiency score. Here, we follow the concept of slacks-based measure (SBM) of efficiency in traditional DEA framework (Tone, 2001) to compute the slacks in inputs and desirable outputs. Therefore, we present the following model:

$$
\begin{aligned}
\rho^* = Min \quad & \frac{1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{ik}}}{1 + \frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{g_{rk}}} \\
s.t. \quad & \sum_{j=1}^{n}(\eta_j + \mu_j)x_{ij} + s_i^- = x_{ik} \qquad (i = 1,\dots,m), \\
& \sum_{j=1}^{n}\eta_j g_{rj} - s_r^+ = g_{rk} \qquad (r = 1,\dots,s), \\
& \sum_{j=1}^{n}\eta_j b_{fj} = \gamma_f^*\, b_{fk} \qquad (f = 1,\dots,h), \\
& \sum_{j=1}^{n}(\eta_j + \mu_j) = 1, \eta_j \ge 0, \mu_j \ge 0 \quad (j = 1,\dots,n), \\
& s_i^- \ge 0, \quad (i = 1,\dots,m), \quad s_r^+ \ge 0, \quad (r = 1,\dots,s).
\end{aligned}
\tag{6}
$$

In fact, model (6) identifies the economical inefficiencies of DMU_k by a slacks-based efficiency measure ρ^* after its undesirable outputs are adjusted to their minimum levels. It can be easily verified that $0 < \rho^* \le 1$,

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and also satisfies the unit invariant property. If there are no slacks in inputs and desirable outputs, i.e. $s_i^{-*} = 0\ (\forall i)$ and $s_r^{+*} = 0\ (\forall r)$, then DMU_k has no economical inefficiencies and attain $\rho^* = 1$.

By merging environmental index *EI* and economical inefficiency ρ^*, the following environmental slacks-based measure (ESBM) is obtained as:

$$ESBM = EI \times \rho^* \tag{7}$$

This measure combines the environmental and economic inefficiencies of DMU_k in a unified manner. Also, it lies in the interval (0, 1] since *EI* and ρ^* are both between zero and unity.

Note that model (6) is a fractional programming problem; however, it can be transformed into an equivalent linear programming problem by using the Charnes-Cooper transformation (Tone, 2001). Putting $1+\frac{1}{s}\sum_{r=1}^{s}\frac{s_r^+}{g_{rk}} = \frac{1}{t}$, and multiplying each constrain by *t*, we then let $t\,\eta_j = \eta'_j, t\,\mu_j = \mu'_j, t\,s_i^- = s'_i$, and $t\,s_r^+ = s'_r$. Therefore, the following linear problem is achieved:

$$\begin{aligned}
\rho^* = Min \quad & t - \frac{1}{m}\sum_{i=1}^{m}\frac{s'_i}{x_{ik}} \\
s.t. \quad & \sum_{j=1}^{n}(\eta'_j + \mu'_j)x_{ij} + s'_i = t\,x_{ik} \qquad (i = 1,\dots,m), \\
& \sum_{j=1}^{n}\eta'_j g_{rj} - s''_r = t\,g_{rk} \qquad (r = 1,\dots,s), \\
& \sum_{j=1}^{n}\eta'_j b_{fj} = t\,\gamma_f^*\, b_{fk} \qquad (f = 1,\dots,h), \\
& \sum_{j=1}^{n}(\eta'_j + \mu'_j) = t,\ \ \eta'_j \ge 0,\ \mu'_j \ge 0 \quad (j = 1,\dots,n), \\
& t + \frac{1}{s}\sum_{r=1}^{s}\frac{s''_r}{g_{rk}} = 1, \\
& s'_i \ge 0,\ \ (i = 1,\dots,m), \quad s''_r \ge 0,\ \ (r = 1,\dots,s).
\end{aligned} \tag{8}$$

As a result, we can easily obtain ρ^* via a linear problem and estimate the ESBM measure for evaluating the environmental performance of DMU_k.

Numerical Example:

In this section, we apply the proposed method for assessing the environmental performance of 31 administrative regions of China. See the data set in table 1 which is adopted form Wu *et al*. article (Wu *et al*, 2013). These data have two inputs: the total investment in the fixed assets of industry (TIFA) and the electricity consumption by industry (EC), one desirable output: the gross industrial output value (GIOV), and two undesirable outputs: the total volume of industrial waste gas emission (TWGE) and the total volume of waste water discharge (TWWD). For ease of comparison, we have named the industries D1 to D31 which is exhibited in the second column of table 1.

Here, the GAMS (General Algebraic Modeling System) software is utilized for the computations. Table 2 displays the results of the presented models. The first column shows the amounts of the environmental Index (EI) measure which is computed by model (5). D2, D6, D16, D23, D24, D27, and D28 are the industries which manage successfully their undesirable outputs and attain *EI*=1. It should be noticed that some industries (such as D7, D8, D15, D20, D21, D25, D29 and D30) obtain a very low environmental index. Therefore, these industries should seriously be concerned about reduction of their pollutants and wastes.

The other columns in table 2 represent respectively the amounts of economical inefficiency ρ^* and environmental slacks-based measure (*ESBM*) which are calculated individually by model (8) and equation (7). The 7 above mentioned industries perform also effectively in their inputs and desirable output and attain $\rho^* = 1$. Thus, we can conclude that these 7 industries pay attention to the reduction of their pollutants accompanying with improving their commercial targets.

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Table 1: Data set of industry of Chian in 2010.

District	TWWD	TWGE	GIOV	EC	TIFA	DMU
Anhui	70971	17849	18732	1077.91	9121.829	D1
Beijing	8198	4750	13699.84	809.9	4554.356	D2
Chongqing	45180	10943	9143.55	626.44	5049.258	D3
Fujian	124168	13507	21901.23	1315.09	6534.803	D4
Gansu	15352	6252	4882.62	804.43	2274.305	D5
Guangdong	187031	24092	85824.64	4060.13	11903.36	D6
Guangxi	165211	14520	9644.13	993.24	5166.135	D7
Guizhou	14130	10192	4206.37	835.38	2483.012	D8
Hainan	5782	1360	1381.25	159.02	903.8264	D9
Hebei	114232	56324	31143.29	2691.52	11737.07	D10
Heilongjiang	38921	10111	9535.15	747.84	5019.085	D11
Henen	150406	22709	34995.53	2353.96	12868.24	D12
Hubei	94593	13865	21623.12	1330.44	7276.638	D13
Hunan	95605	14673	19008.83	1171.91	7374.157	D14
Inner Mongolia	39536	27488	13406.11	1536.83	6831.416	D15
Jiangsu	263760	31213	92056.48	3864.37	18977.92	D16
Jiangxi	72526	9812	13883.06	700.51	6696.149	D17
Jilin	38656	8240	13098.35	576.98	6313.748	D18
Liaoning	71521	26955	36219.42	1715.26	12480.94	D19
Ningxia	21977	16324	1924.39	546.77	1193.702	D20
Qingghai	9031	3952	1481.99	465.18	789.5051	D21
Shaanxi	45487	13510	11199.84	859.22	5462.784	D22
Shandong	208257	43837	83851.4	3298.46	17664.34	D23
Shanghai	36696	12969	30114.41	1295.87	4252.32	D24
Shangxi	49881	35190	12471.33	1460	4702.091	D25
Sichuan	93444	20107	23147.38	1549.03	9790.274	D26
Tianjin	19680	7686	16751.82	645.74	4571.888	D27
Tibet	736	16	62.22	20.41	306.567	D28
Xinjiang	25413	9310	5341.9	5341.9	2749.838	D29
Yunnan	30926	10978	6464.63	1004.07	4024.972	D30
Zhejiang	217426	20434	51394.2	2820.93	10246.41	D31

Table 2: Results of the proposed models.

District	*ESBM*	ρ*	*EI*	DMU
Anhui	0.24	0.759	0.317	D1
Beijing	1	1	1	D2
Chongqing	0.154	0.748	0.205	D3
Fujian	0.32	0.861	0.372	D4
Gansu	0.136	0.588	0.231	D5
Guangdong	1	1	1	D6
Guangxi	0.081	0.609	0.133	D7
Guizhou	0.077	0.482	0.161	D8
Hainan	0.178	0.721	0.248	D9
Hebei	0.127	0.0.437	0.29	D10
Heilongjiang	0.167	0.706	0.237	D11
Henen	0.276	0.637	0.433	D12
Hubei	0.319	0.807	0.395	D13
Hunan	0.255	0.793	0.322	D14
Inner Mongolia	0.109	0.585	0.186	D15
Jiangsu	1	1	1	D16
Jiangxi	0.295	0.82	0.36	D17
Jilin	0.415	0.803	0.516	D18
Liaoning	0.425	0.67	0.635	D19
Ningxia	0.023	0.498	0.047	D20
Qingghai	0.068	0.599	0.114	D21
Shaanxi	0.159	0.733	0.217	D22
Shandong	1	1	1	D23
Shanghai	1	1	1	D24
Shangxi	0.095	0.697	0.136	D25
Sichuan	0.238	0.681	0.35	D26
Tianjin	1	1	1	D27
Tibet	1	1	1	D28
Xinjiang	0.098	0.605	0.162	D29
Yunnan	0.079	0,483	0.165	D30
Zhejiang	0.51	0.854	0.597	D31

Summary and Conclusion:

Nowadays, global warming, climate change, an enhanced emission of CO_2 in the air, and water pollution are major problems all over the world. These worldwide problems reveal the importance of developing firms with less undesirable factors. In this paper, we briefly introduced the two technologies which are available in the DEA literature for modeling environmental performance under weak disposability assumption of desirable and undesirable outputs.

Then, we attempted to present a slacks-based measure that incorporates both desirable and undesirable outputs. This measure is obtained by solving some DEA-based models. First, the undesirable factors are adjusted to their minimum levels, and then all the input excesses and desirable output shortfalls are computed.

To illustrate the use of the proposed method, we applied the proposed method for assessing the environmental performance of 31 administrative regions of China. 7 industries obtained the full environmental slacks-based efficiency measure via the proposed model. Thus, we concluded that these industries pay attention to the reduction of their pollutants accompanying with improving their commercial targets.

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